STAT FOR BIM
LECTURE NOTES

COUNTING RULES;
Permutation & Combination
PROBABILITY
Conditional Prob. & Independence
BAYES' THEOREM

Counting

(i) Basic Principle of Counting

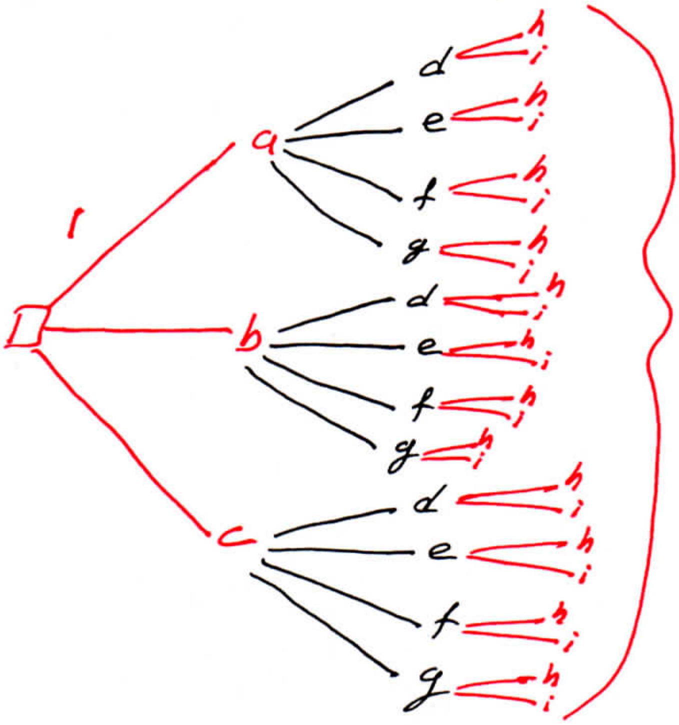
If we are going to select 1 element from the set A_1, 1 element from the set A_2, ..., one from A_k and set A_i has n_i elements, we can make it in

$$n_1 \cdot n_2 \cdot \ldots \cdot n_k \text{ different ways.}$$

Example If there are 3 soups, 4 meals and 2 deserts, in how many ways can one eat a menu one from each?

Let; Soups: $\{a, b, c\}$; Meals; $\{d, e, f, g\}$; Deserts: $\{h, i\}$.

Then; A menu can be formed from the following tree:



24 different ways.

* This is just "Basic Principle of counting"

$$\underset{\text{soups}}{\underline{3}} \cdot \underset{\text{meals}}{\underline{4}} \cdot \underset{\text{deserts.}}{\underline{2}} = 24$$

5.11 In an optics kit there are six concave lenses, four convex lenses, two prisms, and two mirrors. In how many different ways can one choose a concave lens, a convex lens, a prism, and a mirror from this kit?

5.12 A psychologist is preparing three-letter nonsense words for use in a memory test. He chooses the first letter from the consonants k, m, w, and z. He chooses the middle letter from the vowels a, i, and u. He chooses the final letter from the consonants b, d, f, k, m, and t.
(a) How many different three-letter nonsense words can he construct?
(b) How many of these nonsense words will begin with the letter z?
(c) How many of these nonsense words will end with either k or m?
(d) How many of these nonsense words will begin and end with the same letter?

5.11) 6 lenses concave, 4 lenses convex, 2 prisms, 2 mirrors

$\underline{6} \cdot \underline{4} \cdot \underline{2} \cdot \underline{2} = 96$ ways.

5.12) First letter; $A_1 = \{k, m, w, z\}$

Second letter; $A_2 = \{a, i, u\}$

Third letter; $A_3 = \{b, d, f, k, m, t\}$

$n(A_1) = 4$; $n(A_2) = 3$; $n(A_3) = 6$

Then; a) $\underline{4} \cdot \underline{3} \cdot \underline{6} = 72$

b) $\underline{1} \cdot \underline{3} \cdot \underline{6} = 18$

Begin with z, single choice

c) $\underline{4} \cdot \underline{3} \cdot \underline{2} = 24$

$\to$ k OR m, 2 choices

d) Begin and End with k OR m

$\underbrace{\underline{1} \cdot \underline{3} \cdot \underline{1}}_{k} + \underbrace{\underline{1} \cdot \underline{3} \cdot \underline{1}}_{m} = 6$

(ii) Permutation

From a set of n elements if we select and ORDER r elements, we can do this in ${}_nP_r$ different ways

where;

$${}_nP_r = \frac{n!}{(n-r)!} = \underbrace{n \cdot (n-1) \cdot \ldots \cdot (n-r+1)}_{r \text{ numbers.}}$$

This is just basic principle of counting but we have a single set A and for every new order, possible choices decrease by 1.

Example. From a class of 18 students, a President, a Prime Minister and a Member can be selected (in fact, it is an ordering because duties are different) in

$${}_{18}P_3 = \frac{18 \cdot 17 \cdot 16 \cdot 15!}{15!} = 18 \cdot 17 \cdot 16 = 4896 \text{ different ways.}$$

Likewise, in a league of 18 teams, the first three teams can be in 4896 different ways when league finishes.

However, numerical (!) 6/49 does NOT result in ${}_{49}P_6$ because order is NOT important there. We'll see that it is a combination.

5.21 An amusement park has 28 different rides. In how many different ways can a person try four of these rides, assuming that order matters and that she does not want to try any ride more than once?

5.21) ${}_{28}P_4 = 28.27.26.25 = 491400$

(iii) Combination

From a set of n elements we can SELECT r elements in $\binom{n}{r}$ different ways. In other words, a set of n elements have $\binom{n}{r}$ different subsets of r elements, where $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n.(n-1).\ldots.(n-r+1)}{r!}$

Note that, Permulation just "orders" each selection and this can be done in $r!$ different ways. So;

$$r! \cdot \binom{n}{r} = {}_nP_r$$

Example. From a class of 18 students, 3 "identical" members can be selected in $\binom{18}{3} = \frac{18.17.16}{3!} = 816$ ways.

Numerical 6/49 results in $\binom{49}{6} = \frac{49.48.47.46.45.44}{6!}$ $= 13\,983\,816$ different ways. So, if you play a single column, you have the prob. to win big price,

$$\frac{1}{\binom{49}{6}} \cong \text{One in 14 million}$$

5.20 In how many ways can four new corporate clients be assigned to eleven service representatives, assuming that each service representative can be given at most one of the corporate clients?

5.20) $\binom{11}{4} = \frac{11.10.9.8}{4!} = 330$

5.39 A men's clothing story carries eight kinds of sweaters, six kinds of slacks, and ten kinds of shirts. In how many ways can two of each kind be chosen for a special sale?

5.39) 8 sweaters
6 slacks
10 t-shirts
} select 2 from each

In $\binom{8}{2}\cdot\binom{6}{2}\cdot\binom{10}{2} = \frac{8.7}{2!}\cdot\frac{6.5}{2!}\cdot\frac{10.9}{2!} = 18900$ different ways.

Example. Likewise, let there are 8 doctors, 6 nurses and 10 singers, we can select 3 doctors, 5 nurses and 1 singer in $\binom{8}{3}\cdot\binom{6}{5}\cdot\binom{10}{1} = 3360$ different ways.

5.37 A ten-pack of batteries has two defective batteries. In how many ways can one select three of these batteries and get
(a) neither of the defective batteries;
(b) one of the defective batteries;
(c) both of the defective batteries?

5.37) 2 def.
8 NON def
10 items
→ Select n=2

a) $\binom{2}{0}\cdot\binom{8}{2} = 28$ b) $\binom{2}{1}\cdot\binom{8}{1} = 16$ c) $\binom{2}{2}\cdot\binom{8}{0} = 1$

Example. In a class of 15 students, 6 are girls. In how many ways can one select 5 students and get 3 of them girls?

6 girls
9 boys
15 student
→ Select n=5 students

$\underbrace{\binom{6}{3}}_{\text{3 girls out of 6}}\underbrace{\binom{9}{2}}_{\text{2 boys out of 9}} = 720$ different ways.

Example. A poker hand has $\binom{52}{5} = 2598960$ combinations.

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PROBABILITY

* Probability of an event is the relative frequency of its occurance to all possible outcomes. What do we mean by "We have 30% chance to win this match?" The meaning is; "3 out of 10 such difficult matches, we win." So, the probability of an event is;

$$P(\text{Event}) = \frac{\text{Number of outcomes event is occured (can occur)}}{\text{Number of all possible outcomes.}}$$

Consider a deck of 52 cards. We have,

	A	2	3	4	5	6	7	8	9	10	J	Q	K
	NUMBERS										FACES		
♠ Spade (Black)													
♣ Club (Black)													
♢ Diamond (Red)													
♡ Heart (Red)													

5.47 When one card is drawn from a well-shuffled deck of 52 standard playing cards, what are the probabilities of getting
(a) a black queen;
(b) a jack, queen, or king of any suit;
(c) a black card;
(d) any one of a 4, 5, 6, or 7;
(e) a heart?

5.48 Two cards are dealt from a well-shuffled deck. What are the probabilities of getting
(a) two red cards;
(b) two kings?

5.49 If three cards are dealt from a well-shuffled deck, find the probabilities of getting
(a) three spades;
(b) two kings and one queen;
(c) two diamonds and one heart.

5.47) a) $P(\text{Black Queen}) = \frac{2}{52}$

b) $P(\text{Face}) = \frac{12}{52}$ c) $P(\text{Black}) = \frac{26}{52}$

d) $P(4, 5, 6 \text{ OR } 7) = \frac{16}{52}$ e) $P(\text{Heart}) = \frac{13}{52}$

5.48) a) $P(\text{Two Red}) = \frac{\binom{26}{2}}{\binom{52}{2}} = \frac{26}{52} \cdot \frac{25}{51}$

b) $P(\text{Two Kings}) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{4}{52} \cdot \frac{3}{51}$

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5.69) a) $P(\text{Three spades}) = \dfrac{\binom{13}{3}}{\binom{52}{3}} = \dfrac{13}{52}\cdot\dfrac{12}{51}\cdot\dfrac{10}{50}$

b) $P(\text{Two Kings and One Queen}) = \dfrac{\binom{4}{2}\binom{4}{1}}{\binom{52}{3}} = 3\cdot\dfrac{4}{52}\cdot\dfrac{3}{51}\cdot\dfrac{4}{50}$

c) $P(\text{Two Diamonds and one Heart}) = \dfrac{\binom{13}{2}\binom{13}{1}}{\binom{52}{3}} = 3\cdot\dfrac{13}{52}\cdot\dfrac{12}{51}\cdot\dfrac{10}{50}$

Basic Definitions

* **Sample Space:** The set that contains all possible outcomes

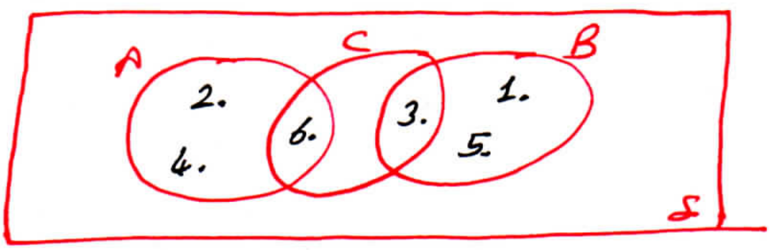
Event: Any subset of Sample Space is an event

Example: let a fair die is tossed. Also let the following events are defined.

A: Outcome is an even number
B: Outcome is an odd number
C: Outcome is multiple of 3.

Then we have; $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2,4,6\}$ $\quad B = \{1,3,5\}$ $\quad C = \{3,6\}$



$P(A) = \frac{3}{6}$; $P(B) = \frac{3}{6}$; $P(C) = \frac{2}{6}$; $P(S) = 1$

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Example; If we toss a fair dice twice, the sample space will be;

$$S = \{(1,1), (1,2), -----, (6,6)\} \; ; \; n(S) = 36$$

II \ I	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

For example, we have

$$P(\text{we have a pair}) = P\{(1,1), (2,2), --, (6,6)\} = \frac{6}{36}$$

$$P\{\text{Sum is } 7\} = P\{(1,6), (2,5), ----, (6,1)\} = \frac{6}{36}$$

Union and intersection of Events;

Union : $\cup$ means "OR" : At least one of the events.

Intersection : $\cap$ means "AND" : Two events together.

$$P(A \cup B) = P(A \text{ OR } B)$$

$$P(A \cap B) = P(A \text{ AND } B) = P(AB)$$

We have; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Rolling a dice twice, define

$A = \{$At least one of the dice is 5$\}$

$B = \{$Sum of the dice is 7$\}$

Then; $P(A) = P\{$First die is 5$\} + P\{$Second die is 5$\}$
$- P\{$Both dice are 5$\}$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$

$P(B) = \frac{6}{36}$; $A \cap B = \{ (5,2), (2,5) \}$

$P(A \text{ OR } B) = P(A \cup B) = \frac{11}{36} + \frac{6}{36} - \frac{2}{36} = \frac{15}{36}$

Complement of an event;

A complement : $\bar{A}$: means ; NOT A ; $\boxed{P(\bar{A}) = 1 - P(A)}$

Rolling two dice, WPT None of them are 5?
what is the probability that

$P\{$None of them are 5$\} = P(\bar{A}) = 1 - P(A) = 1 - \frac{11}{36} = \frac{25}{36}$

Mutually Exclusive Events;

If two events are mutually exclusive, they CANNOT happen together. Then, if A and B are mutually exclusive, $\boxed{P(A B) = 0}$

Note that A and $\bar{A}$ are mutually exclusive events for any event A.

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For example, in "Rolling a dice twice" example;

let $C = \{\text{Sum of the dice is } 11\}$; $P(C) = \frac{2}{36}$

A and B are NOT mutually exclusive because

$A \cap B = \{(5,2), (2,5)\}$

$P(AB) = \frac{2}{36} \neq 0$

B and C are mutually exclusive because

$B \cap C = \emptyset$ and $P(BC) = 0$

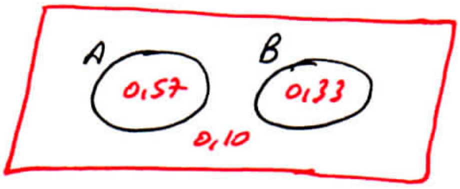
Also, $P(B \cup C) = P(B) + P(C) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36}$

6.42 The probability that a typist will make at most three mistakes when typing a long letter, or make from four to eight, are 0.57 and 0.33. Use the postulates and/or the rules on page 128 to find the probabilities that the typist will make
(a) at least four mistakes;
(b) at most eight mistakes;
(c) more than eight mistakes.

6.39 Given $P(M) = 0.31$ and $P(N) = 0.62$, where M and N are mutually exclusive, use the postulates and/or the rules on page 128 to find
(a) $P(M')$;
(b) $P(N')$;
(c) $P(M \cup N)$;
(d) $P(M' \cap N')$.

6.40 Use a Venn diagram to rework the preceding exercise.

6.42)



$A : \{\text{At most 3 mistakes}\}$; $P(A) = 0,57$

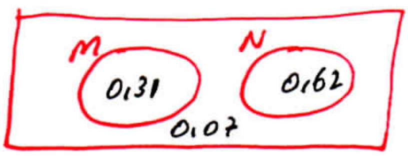
$B : \{\text{From 4 to 8 mistakes}\}$; $P(B) = 0,33$

a) $P(\bar{A}) = 1 - P(A) = 1 - 0,57 = 0,43$

b) $P(A \cup B) = P(A) + P(B) = 0,57 + 0,33 = 0,90$

c) $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0,90 = 0,10$

6.39-6.40)



a) $P(\bar{M}) = 1 - P(M) = 1 - 0,31 = 0,69$

b) $P(\bar{N}) = 1 - P(N) = 1 - 0,62 = 0,38$

c) $P(M \cup N) = P(M) + P(N) = 0,31 + 0,62 = 0,93$

d) By De-Morgan's Rule; we have $\overline{A \cup B} = \bar{A} \cap \bar{B}$ and
$\overline{A \cap B} = \bar{A} \cup \bar{B}$

So; $P(\bar{M} \cap \bar{N}) = P(\overline{M \cup N}) = 1 - P(M \cup N) = 1 - 0,93 = 0,07$

6.64 Figure 6.11 pertains to the number of persons who are invited to a conference and the number of persons who attend. If each of the 35 points of the sample space has the probability $\frac{1}{35}$, what are the probabilities that
(a) at most three persons will attend;
(b) at least six persons will be invited;
(c) one invited person will not attend?

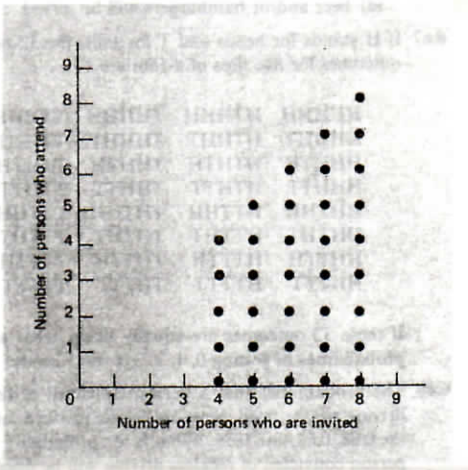


FIGURE 6.11 *Sample space for Exercise 6.64.*

6.64) a) P(At most 3 persons attend)

$= 20 \cdot \frac{1}{35} = \frac{20}{35}$

b) P(At least 6 persons invited)

$= 6 \cdot \frac{1}{35} = \frac{6}{35}$

c) P(One invited NOT attend)

$= 5 \cdot \frac{1}{35} = \frac{5}{35}$

6.70 The probabilities that a person stopping at a gas station will ask to have his tires checked is 0.14, the probability that he will ask to have his oil checked is 0.27, and the probability that he will ask to have them both checked is 0.09. What are the probabilities that a person stopping at this gas station will have
(a) his tires, his oil, or both checked;
(b) neither his tires nor his oil checked?

6.70) P(Tires) = 0,14

P(Oil) = 0,27

P(Tires AND Oil) = 0,09

P(Tires OR Oil) = 0,14 + 0,27 − 0,09 = 0,32

Conditional Probability & Independence

* Basically, there are 3 types of probability;

(i) Marginal → Single Event
(ii) Joint → Two Events Together
(iii) Conditional → Event given that another event has occured

EX) A study on color choices and gender is shown in the following table:

Gender \ Color	Pink	Blue	White	TOTAL
Male	6	16	8	40
Female	35	15	10	60
TOTAL	41	31	18	100

what is the probability that;

a) A randomly choosen person is male?

b) A randomly choosen person selects White?

c) A randomly choosen person is female and selects Blue?

d) If it is known that color choice is Pink, WPT the person is female?

e) WPT a female chooses White?

Answer; (i) Marginal

a) $P(Male) = \frac{40}{100}$ b) $P(White) = \frac{18}{100}$

(ii) Joint

c) $P(Female, Blue) = \frac{15}{100}$

(iii) Conditional

d) $P(Female \mid Pink) = \frac{35}{41}$ e) $P(White \mid Female) = \frac{10}{60}$

* Consider the probability in part (e)

$$P(W \mid F) = \frac{10}{60} = \frac{10/100}{60/100} = \frac{P(W\cancel{F})}{P(F)}$$

In General; $\boxed{P(A \mid B) = \frac{P(AB)}{P(B)}}$

Given

$P(A \mid B)$: Probability of A given B

Random event in (the knowledge of) B

Known Event

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Multiplication Rule

We have by cross multiplication;

$$\boxed{P(AB) = P(B) \cdot P(A|B)}$$ → Multiplication Rule for any Events A and B

For example; $P(FB) = P(F) \cdot P(B|F) = \frac{60}{100} \cdot \frac{15}{60} = \frac{15}{100}$

* Two events A and B are independent if knowledge of occurance of one event does NOT change the probability of the other event.

Then; A and B are independent if;

$$\boxed{P(A|B) = P(A)} \quad (\text{or } P(B|A) = P(B))$$

We have;

$P(A|B) = P(A)$ → only for independent events

$$\frac{P(AB)}{P(B)} = P(A)$$

$$\boxed{P(AB) = P(A) \cdot P(B)}$$ → Multiplication Rule for INDEPENDENT events A and B

Ex: Is Gender and Color choice independent?

Answer: Check if $P(\text{Color Choice}, \text{Gender}) = P(\text{Color Choice}) \cdot P(\text{Gender})$ for every choice of Color Choice and Gender.

$$\underset{6/100}{P(M \cap P)} \stackrel{?}{=} \underset{40/100}{P(M)} \cdot \underset{41/100}{P(P)}$$

$0{,}06 \neq 0{,}164$ → NOT independent

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Example. Recall the deck of 52 cards example. WPT;

a) a randomly choosen card is Face and Black?

b) a Face is Black?

c) a Black card is Face?

d) are Face and Black independent?

e) are Face and Black mutually exclusive?

f) are Hearts and Black independent?

g) are Hearts and Black mutually exclusive?

Answer

a) $P(\text{Face Black}) = \frac{6}{52}$

b) $P(\text{Black} \mid \text{Face}) = \frac{6}{12}$

c) $P(\text{Face} \mid \text{Black}) = \frac{6}{26}$

d) $P(\text{Face}) . P(\text{Black}) = \frac{\cancel{12}^{\,6}}{52} . \frac{26}{\cancel{52}_{\,2}} = \frac{6}{52} = P(\text{Face Black})$

Yes, they are independent $[P(AB) \overset{?}{=} P(A) . P(B)]$

e) $P(\text{Face Black}) \neq 0$ No, NOT mutually exclusive $[P(AB) \overset{?}{=} 0]$

f) $P(\text{Hearts Black}) = 0 \neq P(\text{Hearts}) . P(\text{Black})$

No, they are NOT independent

g) $P(\text{Hearts Black}) = 0$

Yes, they are mutually exclusive

6.78 A guidance department gives students various kinds of tests. If I is the event that a student scores high in intelligence, A is the event that a student rates high on a social adjustment scale, and N is the event that a student displays neurotic tendencies, express symbolically the probabilities that

(a) a student who scores high in intelligence will display neurotic tendencies;

(b) a student who does not rate high on the social adjustment scale will not score high in intelligence;

(c) a student who displays neurotic tendencies will neither score high in intelligence nor rate high on the social adjustment scale.

6.79 With reference to the preceding exercise, state in words what probabilities are expressed by

(a) $P(I|A)$;
(b) $P(A'|N')$;
(c) $P(N'|I \cap A)$.

6.78) I : Intelligence
A : Social Adjustment
N : Neurotic Tendencies.

a) $P(N|I)$ b) $P(\bar{I}|\bar{A})$ c) $P(\overline{I \cup A}|N)$

THE PROBABILITY THAT;

6.79) a) a student who scores high in social adjustment will score high in intelligence

b) a student who does NOT display neurotic tendencies will NOT score high in social adjustment

c) a student who got high in both intelligence and social adjustment will NOT display neurotic tendencies.

6.82 There are 80 applicants seeking to obtain a fast food franchise. Some of these persons are college graduates and some are not. Some have prior experience in the food service industry and some do not. The exact breakdown is

	G: College graduates	Not college graduates : $\bar{G}$
E: Prior food service experience	24	36
$\bar{E}$: No prior food service experience	12	8

If the order in which the applicants are processed is random, G is the event that the first applicant processed is a college graduate, and E is the event that the first applicant processed has prior food service experience, determine each of the following probabilities directly from the entries and the row and column totals of the table:

(a) $P(G)$;
(b) $P(E')$;
(c) $P(G \cap E)$;
(d) $P(G' \cap E')$;
(e) $P(E|G)$;
(f) $P(G'|E')$;
(g) $P(E'|G')$;
(h) $P(E \cup G)$.

6.82)

	G	$\bar{G}$	Total
E	24	36	60
$\bar{E}$	12	8	20
Total	36	44	80

a) $P(G) = \frac{36}{80}$

b) $P(\bar{E}) = \frac{20}{80}$

c) $P(GE) = \frac{24}{80}$

d) $P(\bar{G}\bar{E}) = \frac{8}{80}$

e) $P(E|G) = \frac{24}{36}$

f) $P(\bar{G}|\bar{E}) = \frac{8}{20}$

g) $P(\bar{E}|\bar{G}) = \frac{8}{44}$

h) $P(E \cup G) = 1 - P(\overline{EG})$
$= 1 - \frac{8}{80} = \frac{72}{80}$

6.88 The probability that a bus from Cleveland to Chicago will leave on time is 0.80, and the probability that it will leave on time and also arrive on time is 0.72.

(a) What is the conditional probability that if such a bus leaves on time it will also arrive on time?

(b) If the probability is 0.75 that such a bus will arrive on time, what is the conditional probability that if such a bus does not leave on time it will nevertheless arrive on time?

6.91 The probability that a student attending a Western college will buy a personal computer is 0.50, and the probability that if he buys a personal computer his grades will go up is 0.72. What is the probability that a student attending this college will buy a personal computer and have his grades go up?

6.88) $P(L) = 0{,}80$; $P(LA) = 0{,}72$

a) $P(A \mid L) = \dfrac{P(AL)}{P(L)} = \dfrac{0{,}72}{0{,}80} = 0{,}9$

b) $P(A) = 0{,}75 \Rightarrow P(\bar{A} \mid \bar{L}) = ?$

$P(A \cup L) = P(A) + P(L) - P(AL)$

$P(A \cup L) = 0{,}75 + 0{,}80 - 0{,}72 = 0{,}83$

$P(\bar{A} \mid \bar{L}) = \dfrac{P(\bar{A}\,\bar{L})}{P(\bar{L})} = \dfrac{P(\overline{A \cup L})}{P(\bar{L})}$

$= \dfrac{1 - P(A \cup L)}{1 - P(L)} = \dfrac{1 - 0{,}83}{1 - 0{,}80} = \dfrac{0{,}17}{0{,}20} = 0{,}85$

6.91) $P(Buy) = 0{,}50$

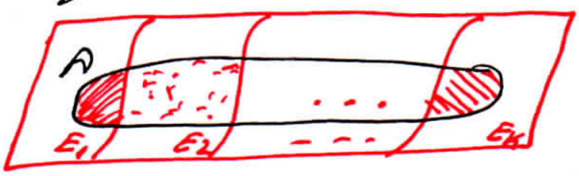
$P(Up \mid Buy) = 0{,}72$

$P(Up\ Buy) = P(Buy) \cdot P(Up \mid Buy)$

$= 0{,}50 \cdot 0{,}72 = 0{,}36$

Total Probability Rule & Bayes' Theorem

Let $E_1, E_2, \dots, E_k$ are Mutually Exclusive events and Their union is Sample space. (Like Departments in Bilkent University). Let, A be another event.



$P(A) = P(AE_1) + P(AE_2) + \dots + P(AE_k)$

$$\boxed{P(A) = P(E_1)\,P(A \mid E_1) + P(E_2)\,P(A \mid E_2) + \dots + P(E_k)\,P(A \mid E_k)}$$

Total Probability Rule

Then;

$$\boxed{P(E_j \mid A) = \frac{P(E_j A)}{P(A)} = \frac{P(E_j)\,P(A \mid E_j)}{\sum_{i=1}^{k} P(E_i)\,P(A \mid E_i)}}$$

Bayes' Theorem

* Also Remember; B and $\bar{B}$ are also mutually exclusive events whose union is Sample space. Then;

$$P(A) = P(B) . P(A|B) + P(\bar{B}) . P(A|\bar{B})$$

$$\boxed{P(B|A) = \frac{P(BA)}{P(A)} = \frac{P(B)\,P(A|B)}{P(B)P(A|B) + P(\bar{B})P(A|\bar{B})}}$$

6.116 It is known from experience that in a certain industry 60 percent of all labor–management disputes are over wages, 15 percent are over working conditions, and 25 percent are over fringe issues. Also, 45 percent of the disputes over wages are resolved without strikes, 70 percent of the disputes over working conditions are resolved without strikes, and 40 percent of the disputes over fringe issues are resolved without strikes.
a) What is the probability that a labor–management dispute in this industry will be resolved without a strike?

6.116) Let, E_1 : Over wages
E_2 : Over working Condition
E_3 : Over fringe Issues
R : Resolved without strikes.

We have;

$P(E_1) = 0{,}60 ; P(R|E_1) = 0{,}45$
$P(E_2) = 0{,}15 ; P(R|E_2) = 0{,}70$
$P(E_3) = 0{,}25 ; P(R|E_3) = 0{,}40$

$$P(R) = P(E_1)\,P(R|E_1) + P(E_2)P(R|E_2) + P(E_3)\,P(R|E_3)$$
$$= 0{,}60 . 0{,}45 + 0{,}15 . 0{,}70 + 0{,}25 . 0{,}40 = 0{,}475$$

b) If a dispute is resolved without strike, WPT it was because of over working conditions?

$$P(E_2|R) = \frac{0{,}15 . 0{,}70}{0{,}60 . 0{,}45 + 0{,}15 . 0{,}70 + 0{,}25 . 0{,}40} = 0{,}221$$

6.113 In a certain community, 8 percent of all adults over 50 have diabetes. If a doctor in this community correctly diagnoses 95 percent of all persons with diabetes as having the disease and incorrectly diagnoses 2 percent of all persons without diabetes as having the disease, what is the probability that an adult over 50 diagnosed by this doctor as having diabetes actually has the disease?

$P(A) = 0{,}08 ; P(D|A) = 0{,}95$
$P(\bar{A}) = 0{,}92 ; P(D|\bar{A}) = 0{,}02$

$$P(A|D) = \frac{0{,}08 . 0{,}95}{0{,}08 . 0{,}95 + 0{,}92 . 0{,}02} = \underline{\underline{0{,}805}}$$

A: Actually has the disease ; D: Diagnoses