STAT FOR BIM LECTURE NOTES

Random Variables
Probability Distributions
Mean and Variance
Discrete Random Variables
Continuous Random Variables

Random Variables

Discrete

* X takes Numbers

Ex// Number of people who supports Galatasaray among 80 randomly selected people

Number of children a randomly selected family has

...etc.

Continuous

* X takes values over an interval.

Ex// weight of a newborn baby

weekly food expenditure of a student

...etc.

DISCRETE RANDOM VARIABLES

Let $f(x)$ be given. $f(x)$ must satisfy the following conditions to be a "probability mass function" (pmf) of a discrete random variable: X

(i) $f(x) \geq 0$ → Nonnegative Probabilities

(ii) $\sum_{x} f(x) = 1$ → Total probability is 1.

Also, for discrete case, we have

$$P(X = x) = f(x)$$

→ The function $f(x)$ gives the probability at $X=x$.

Random Variable ← X; x → Numerical Value

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Examples;

(i) $f(x) = \frac{1}{5}$ for $x = 0, 1, 2, 3, 4, 5$ is NOT a pmf because

$$\sum_{x=0}^{5} f(x) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{6}{5} \neq 1$$

(ii) $f(x) = \frac{x-2}{5}$ for $x = 1, 2, 3, 4, 5$ is NOT a pmf because

$$f(1) = \frac{1-2}{5} = \frac{-1}{5} < 0$$

(iii) $f(x) = \frac{x^2}{30}$ for $x = 0, 1, 2, 3, 4$ is a pmf.

X	0	1	2	3	4
$P(X=x)$	0	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

; $f(x) \geq 0$ and

$$\sum_{x=0}^{4} f(x) = 0 + \frac{1}{30} + \frac{4}{30} + \frac{9}{30} + \frac{16}{30} = \frac{30}{30} = 1$$

(iv) If;

X	0	1	2	3
$f(x)$	0,1	0,4	0,3	?

is a pmf,

$$f(3) = P(X=3) = 1 - [0,1 + 0,4 + 0,3] = 0,2$$

Example. Let, X be the number of cars sold in a gallery is given by;

X	0	1	2	3	4	5	6
$P(x)$	0,05	0,10	0,20	0,20	0,20	0,15	0,10

Then; $P(3 \leq X < 6) = P(3) + P(4) + P(5) = 0,20 + 0,15 + 0,10 = 0,55$

$P(X > 3) = P(4) + P(5) + P(6) = 0,20 + 0,15 + 0,10 = 0,45$

$P(X \leq 4) = 1 - P(X > 4) = 1 - [P(5) + P(6)] = 1 - [0,15 + 0,10] = 0,75$

$P(2 < X \leq 5) = P(3 \leq X < 6) = 0,55$

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* Also note the following;

WPT; X is more than 3 → $P(X>3)=1-P(X\leq 3)$

X is at least 3 → $P(X\geq 3)=1-P(X\leq 2)$

X is less than 3 → $P(X<3)=P(X\leq 2)$

X is at most 3 → $P(X\leq 3)$

Expected Value, Variance & Standard Deviation

* Note that, "Expected Value", "Average" and "Mean" has the same meaning. Expected Value of a Random Variable is its long term average.

We have;

$$\boxed{\mu = E(X) = \sum_x x . f(x)}$$

To find Variance, we need $E(X^2) = \sum_x x^2 . f(x)$

$$\boxed{\sigma^2 = Var(X) = E(X^2) - \mu^2} = E[(X-\mu)^2]$$

σ: Standard deviation is; $\boxed{\sigma = \sqrt{\sigma^2}}$

7.11 A police chief knows that the probabilities of 0, 1, 2, 3, or 4 burglaries on any given day are 0.12, 0.25, 0.39, 0.18, and 0.06. How many burglaries can the police chief expect per day? It is assumed here that the probability of more than 4 burglaries is negligible.

7.11) X: Number of Burglaries the police chief struggles on a given day

X	0	1	2	3	4
f(x)	0,12	0,25	0,39	0,18	0,06

$$\mu = E(X) = \sum_{x=0}^{4} x . f(x) = 0.0,12 + 1.0,25 + 2.0,39 + 3.0,18 + 4.0,04 = \underline{1,73}$$

b) Find the standard Deviation of Number of Burglaries.

$$E(X^2) = \sum_{x=0}^{4} x^2 . f(x) = 0^2.0,12 + 1^2.0,25 + 2^2.0,39 + 3^2.0,18 + 4^2.0,04 = 4,07$$

$$\sigma^2 = 4,07 - 1,73^2 = 1,077 \; ; \; \sigma = \sqrt{1,077} = \underline{1,038}$$

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Important Discrete Distributions

* Each distribution has its own characteristics. We should learn them to identify "which distribution to use to solve the question"

* We use formulas for mean and variance for each specific distribution, do NOT calculate from basic definition we have learnt.

* A distribution is characterized by its parameters (i), pmf (ii), mean (iii) and standard deviation (iv)

(I) The Binomial Distribution

$X \sim \text{Binomial}(n; p)$

where n: Number of independent trials
p: Fixed probability of success
X: Number of success

$$P(X=x) = f(x) = \binom{n}{x} \cdot p^x (1-p)^{n-x}$$

$$\mu = E(X) = n \cdot p \qquad \sigma^2 = Var(X) = n \cdot p \cdot (1-p)$$

* Remember; $\binom{n}{x} = \frac{n!}{x!(n-x)!} = \frac{n \cdot (n-1) \cdot \ldots \cdot (n-x+1)}{x!}$

Example; A basketball player scores 43% of his 3-point shots. If he makes 12 shots in a game, WPT he scored 7 of them?

$X \sim \text{Binomial}(n=12; p=0{,}43)$

$P(X=7) = f(7) = \binom{12}{7} \cdot 0{,}43^7 \cdot 0{,}57^{12-7} = \underline{0{,}13}$

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8.15 It is known that 20 percent of all persons given a certain medication get very drowsy within two minutes. Find the probabilities that among fourteen persons given the medication
(a) at most two will get very drowsy within two minutes;
(b) at least five will get very drowsy within two minutes;
(c) two, three, or four will get very drowsy within two minutes.

8.15) $X \sim Binomial(n=14; p=0{,}20)$

$$f(x) = \binom{14}{x} \cdot 0{,}2^{x} \cdot 0{,}8^{14-x}$$

a) $P(X \leq 2) = f(2) + f(1) + f(0)$

$$= \binom{14}{2} 0{,}2^{2} \cdot 0{,}8^{12} + \binom{14}{1} 0{,}2^{1} \cdot 0{,}8^{13} + \binom{14}{0} 0{,}2^{0} \cdot 0{,}8^{14} = \underline{\underline{0{,}448}}$$

b) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - [f(4) + f(3) + f(2) + f(1) + f(0)]$

$$= 1 - 0{,}87 = \underline{\underline{0{,}13}}$$

c) $P(2 \leq X \leq 4) = f(2) + f(3) + f(4) = \underline{\underline{0{,}672}}$

d) Find mean and std-dev of persons who get very drowsy within two minutes.

$$\mu = E(X) = n \cdot p = 14 \cdot 0{,}2 = \underline{\underline{2{,}8}}$$

$$\sigma^2 = Var(X) = n \cdot p \cdot (1-p) = 14 \cdot 0{,}2 \cdot 0{,}8 = 2{,}24$$

$$\sigma = \sqrt{2{,}24} = \underline{\underline{1{,}5}}$$

(II) The Geometric Distribution

$X \sim Geometric(p)$

p: Fixed Probability of success

X: # of trials to obtain FIRST success

$$f(x) = p \cdot (1-p)^{x-1}$$

$$\mu = E(X) = \frac{1}{p} \quad ; \quad \sigma^2 = Var(X) = \frac{1-p}{p^2}$$

(a) When taping a television commercial, the probability that a certain actor will get his lines straight on any one take is 0.40. What is the probability that this actor will get his lines straight for the first time on the fourth take?

(b) Suppose the probability is 0.25 that any given person will believe a rumor about the private life of a certain politician. What is the probability that the fifth person to hear the rumor will be the first one to believe it?

(c) The probability is 0.70 that a child exposed to a certain contagious disease will catch it. What is the probability that the third child exposed to the disease will be the first one to catch it?

a) $X \sim Geometric\ (p = 0{,}40)$

$P(X=4) = 0{,}40 \cdot 0{,}60^3 = 0{,}0864$

b) $X \sim Geometric\ (p = 0{,}25)$

$P(X=5) = 0{,}25 \cdot 0{,}75^4 = 0{,}0791$

b′) Find expected Number of People and std-dev to find first one to believe it.

$\mu = \frac{1}{0{,}25} = 4\ ;\quad \sigma^2 = \frac{0{,}75}{0{,}25^2} = 12;\ \sigma = \sqrt{12} = 3{,}64$

c) $X \sim Geometric\ (p = 0{,}70)$

$P(X=3) = 0{,}70 \cdot 0{,}30^2 = 0{,}063$

(III) The Hypergeometric Distribution

$\boxed{\begin{matrix} a: \text{Type I items} \\ b: \text{Type II items} \end{matrix}}$ → Select n items randomly

Total: $a+b$ items

X = Number of Type I items.

$$X \sim Hypergeometric\ (a, b, n)$$

$$f(x) = \frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$$

$$\mu = E(X) = n \cdot \frac{a}{a+b}\ ;\quad \sigma^2 = Var(X) = n \cdot \frac{a}{a+b} \cdot \frac{b}{a+b} \cdot \frac{a+b-n}{a+b-1}$$

8.25 Among the 20 solar collectors on display at a trade show, 12 are flat-plate collectors and the others are concentrating collectors. If a person visiting the show randomly selects six of the solar collectors to check out, what is the probability that ~~two~~ of them will be flat-plate collectors?

8.25) $\boxed{\begin{matrix} 12 \text{ flat} \\ 8 \text{ NOT} \end{matrix}}$ → Select $n=6$ connectors.

20 solar con.

X: # of flat-plate connectors

$$P(X=2) = \frac{\binom{12}{2}\binom{8}{4}}{\binom{20}{6}} = 0{,}119$$

(IV) The Poisson Distribution

$X \sim \text{Poisson}(\mu)$

μ: Average Number of Events (or $\mu = n.p$)

X: Number of Events (or success)

$$f(x) = \frac{e^{-\mu} \cdot \mu^x}{x!}$$

$E(X) = \mu \qquad Var(X) = \mu$

Poisson distribution has 2 types of questions;

(i) Only average is known and we want to find the probability of Number of "events occured" (or success)

(ii) In fact, X has a Binomial Distribution but n is too high and p is close to 0.

8.36 If 0.6 percent of the fuses delivered to an arsenal are defective, use the Poisson approximation to the binomial distribution to determine the probability that in a random sample of 500 fuses, four will be defective.

8.42 The number of inquiries a person gets in response to a newspaper ad listing a piano for sale is a random variable having the Poisson distribution with $\lambda = 4.4$. What are the probabilities that in response to such an ad a person will receive
(a) only two inquiries;
(b) only three inquiries;
(c) at most three inquiries?

8.36) $X \sim \text{Binomial}(n = 500 ; p = \overset{0.6\%}{0{,}006})$

$\mu = n.p = 500 . 0{,}006 = 3$

$X \underset{approx.}{\sim} \text{Poisson}(\mu = 3)$

$$P(X=4) = f(4) = \frac{e^{-3} \cdot 3^4}{4!} = 0{,}168$$

8.42) $X \sim \text{Poisson}(\mu = 4{,}4)$

$$f(x) = \frac{e^{-4{,}4} \cdot 4{,}4^x}{x!}$$

a) $P(X=2) = \dfrac{e^{-4{,}4} \cdot 4{,}4^2}{2!} = 0{,}119$

b) $P(X=3) = \dfrac{e^{-4{,}4} \cdot 4{,}4^3}{3!} = 0{,}174$

c) $P(X \leq 3) = f(3) + f(2) + f(1) + f(0)$

$= e^{-4{,}4}\left(\dfrac{4{,}4^3}{3!} + \dfrac{4{,}4^2}{2!} + \dfrac{4{,}4^1}{1!} + \dfrac{4{,}4^0}{0!}\right)$

$= 0{,}359$

(V) The Negative Binomial Distribution

$X \sim$ Negative Binomial $(r; p)$

p: Fixed probability of success

X: Number of trials to obtain r^{th} success

$$f(x) = \binom{x-1}{r-1} \cdot p^r \cdot (1-p)^{x-r}$$

$$\mu = E(X) = \frac{r}{p} \; ; \; \sigma^2 = Var(X) = \frac{r \cdot (1-p)}{p^2}$$

* Note that when $r=1$ (FIRST success), we have Geometric distribution.

Example. If 60% of the employees have positive indications of asbestos in their lungs, WPT ten employees must be tested to find three positives?

Ans $X \sim$ Negative Binomial $(r=3; p=0{,}4)$

$$P(X=10) = \binom{9}{2} \cdot 0{,}4^3 \cdot 0{,}6^7 = \underline{\underline{0{,}0645}}$$

VI The Discrete Uniform Distribution

$X \sim$ Uniform (k)

k: Number of "Equally Likely" Numbers (or categories)

$$f(x) = \frac{1}{k} \; ; \; \mu = E(X) = \frac{k+1}{2} \; ; \; \sigma^2 = Var(X) = \frac{(k-1)(k+1)}{12}$$

Example. Rolling a fair die : $X \sim$ Uniform $(k=6)$

$$f(x) = \frac{1}{6}, \; x = 1, 2, \ldots, 6 \; ; \; \mu = \frac{7}{2} = 3{,}5 \; ; \; \sigma^2 = \frac{5 \cdot 7}{12} = 2{,}917$$

CONTINUOUS RANDOM VARIABLES

Let $f(x)$ be given for $\alpha < x < \beta$. $f(x)$ must satisfy the following conditions to be a "probability density function" (pdf) of a continuous random variable: X

(i) $f(x) \geq 0$ → Nonnegative Probabilities

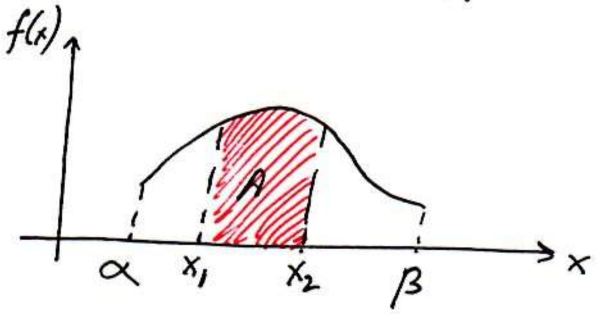
(ii) $\int_{\alpha}^{\beta} f(x)dx = 1$ → TOTAL AREA under the curve is 1.

(We will NOT learn integration. The only thing you should know is that we find areas under a curve (probability) by integral.)

Also, for continuous case; We have,

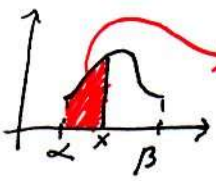
$P(X=x) = 0$ → The probability that X is equal to "any number" is 0.

$P(x_1 < X < x_2) = \int_{x_1}^{x_2} f(x)dx$ → The probability that X is between two Numbers is the area between them and under the curve $f(x)$



$$\left.\begin{array}{l} A = P(x_1 < X < x_2) \\ A = P(x_1 \leq X \leq x_2) \end{array}\right\}$$ "Equality" has NO importance for Continuous Case.

* $F(x)$: Cummulative Distribution Function



$F(x) = P(X \leq x)$ → We'll find the probabilities by $F(x)$ so, we won't take integral.

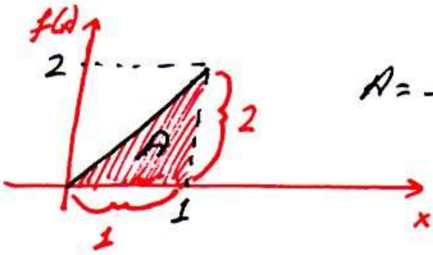
(X: Random variable; x: Numerical value)

$P(x_1 < X < x_2) = P(X < x_2) - P(X < x_1) = F(x_2) - F(x_1)$

$P(X > x_1) = 1 - P(X \leq x_1) = 1 - F(x_1)$

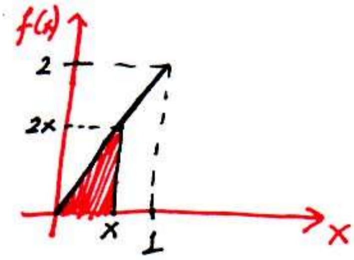
Example. Let $f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

$f(x)$ is a pdf because,



$A = \frac{1.2}{2} = 1$

and $f(x) \ge 0$ for $0 < x < 1$



$F(x) = \underbrace{\frac{x \cdot 2x}{2}}_{\text{Area of Triangle}} = x^2 \quad 0 \le x \le 1$

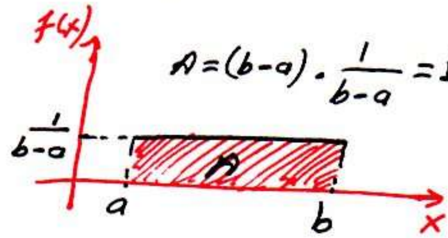
Then; for example,

- $P(X < 0{,}7) = F(0{,}7) = 0{,}7^2 = 0{,}49 = P(X \le 0{,}7)$
- $P(0{,}2 \le X \le 0{,}5) = F(0{,}5) - F(0{,}2) = 0{,}5^2 - 0{,}2^2 = 0{,}25 - 0{,}04 = 0{,}21$
- $P(X \ge 0{,}6) = 1 - P(X < 0{,}6) = 1 - F(0{,}6) = 1 - 0{,}6^2 = 0{,}64$
- $P(X < 10) = 1 \rightarrow$ Since ALL the area is covered by $X < 10$
- $P(X = 0{,}8) = 0 \rightarrow$ Since X is continuous

Important Continuous Distributions

(I) The Continuous Uniform Distribution

$X \sim \text{Uniform}(a; b)$



$A = (b-a) \cdot \frac{1}{b-a} = 1$

$f(x) = \frac{1}{b-a}$ for $a \le x \le b$

$F(x) = \frac{x-a}{b-a}$ for $a \le x \le b$

*All equal length intervals are equally likely.

$\mu = E(X) = \frac{b+a}{2}$ $\quad \sigma^2 = Var(X) = \frac{(b-a)^2}{12}$

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Example. You are waiting for a bus and you don't have a watch. Busses arrive 15 minute interval. Let X be your waiting time of the bus. WPT you wait

a) less than 7 minutes

b) More than 10 minutes

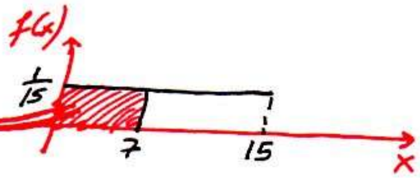
c) Between 5 to 9 minutes

d) Find mean and standard deviation of your waiting time.

Answer $X \sim \text{Uniform}(\underset{a}{0} ; \underset{b}{15})$

$$F(x) = \frac{x - 0}{15 - 0} = \frac{x}{15}$$

a) $P(X < 7) = F(7) = \frac{7}{15}$



b) $P(X > 10) = 1 - P(X \leq 10) = 1 - F(10) = 1 - \frac{10}{15} = \frac{5}{15}$

c) $P(5 < X < 9) = P(X < 9) - P(X < 5) = F(9) - F(5) = \frac{9}{15} - \frac{5}{15} = \frac{4}{15}$

d) $\mu = E(X) = \frac{b+a}{2} = \frac{15+0}{2} = 7{,}5$ minutes.

$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(15-0)^2}{12} = 18{,}75 \Rightarrow \sigma = \sqrt{18{,}75} = 4{,}33$ minutes.

(II) The Exponential Distribution

$X \sim \text{Exponential}(\lambda)$

$f(x) = \lambda . e^{-\lambda x}$

$F(x) = 1 - e^{-\lambda x}$

$E(X) = \frac{1}{\lambda}$; $Var(X) = \frac{1}{\lambda^2}$

λ: Rate of events (Average Number of Events per unit time or unit distance etc..)

Example. There are on the average 2 cars per km. on a highway. The distance between cars have an Exponential distribution with rate 2. WPT the distance between two cars is,

a) Less than 3 km?
b) At least 5 kms?
c) Between 1 to 4 kms?
d) Find mean and standard deviation of the distance between cars.

Answer $X \sim Exponential(\lambda=2)$

$$F(x) = 1 - e^{-2x}$$

a) $P(X < 3) = F(3) = 1 - e^{-2 \cdot 3} = 1 - e^{-6} = \underline{0,998}$

b) $P(X \geq 5) = 1 - P(X < 5) = 1 - F(5) = 1 - (1 - e^{-2 \cdot 5})$
$= e^{-10} = \underline{0,0000454}$

c) $P(1 < X < 4) = F(4) - F(1) = (1 - e^{-4 \cdot 2}) - (1 - e^{-1 \cdot 2})$
$= e^{-2} - e^{-8} = \underline{0,135}$

d) $E(X) = \frac{1}{2} = 0,5$ km; $\sigma^2 = \frac{1}{4} = 0,25 \Rightarrow \sigma = \sqrt{0,25} = 0,5$ km.

Example. A salesman can sell 3 insurance policy per month on the average. WPT he will not sell any policy next week?

Answer: X: Time between policy sales
$X \sim Exponential(\lambda = 3 / month)$

$P(X > \frac{1}{4}) = 1 - P(X \leq \frac{1}{4}) = 1 - F(\frac{1}{4}) = 1 - (1 - e^{-3 \cdot \frac{1}{4}}) = e^{-3/4} = 0,4724$

(III) The Normal Distribution

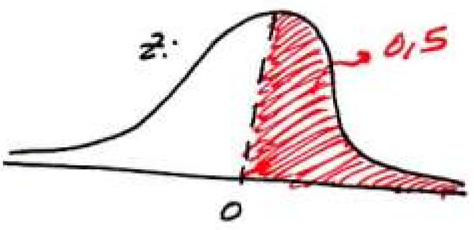
$X \sim Normal\,(\mu\,;\sigma^2)$

$E(X) = \mu \qquad Var(X) = \sigma^2$

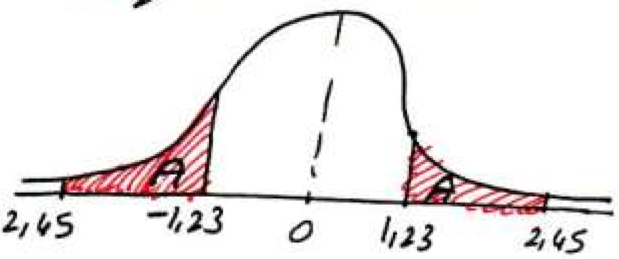
(We do NOT need $f(x)$ or $F(x)$ because we use z-table to find probabilities about X)

The Standard Normal Distribution

$$Z = \frac{X-\mu}{\sigma} \;;\quad Z \sim Normal\,(\mu = 0\,;\sigma^2 = 1^2)$$

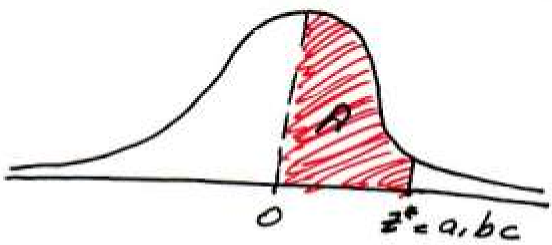


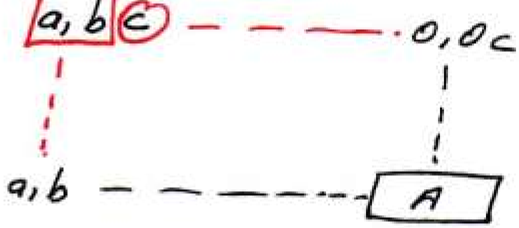
We have
(i) Total AREA = 1
(ii) Half AREA = 0,5
(iii) Symmetric AREA's are equal



$P(-2{,}45 < Z < -1{,}23) = P(1{,}23 < Z < 2{,}45)$

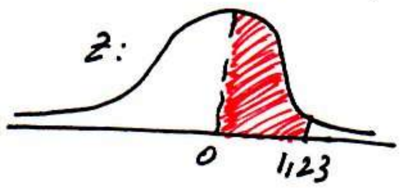
Finding Probabilities from Z-table





$A = P(0 < Z < a{,}bc)$

(I) $P(0 < z < 1,23) = ?$



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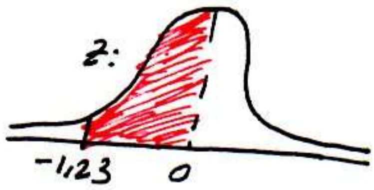
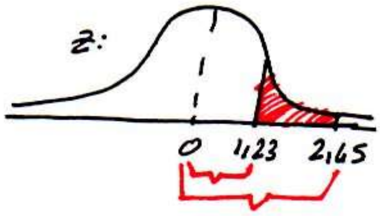


$P(0 < z < 1,23) = 0,3907 = P(-1,23 < z < 0)$

(II) $P(1,23 < z < 2,45) = ?$



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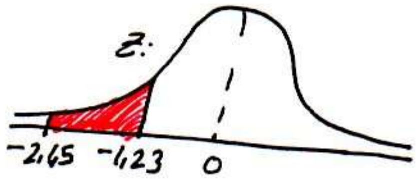
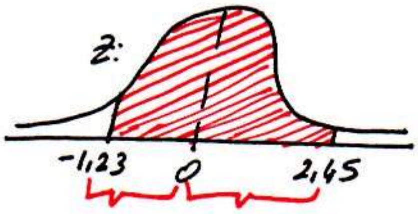


$P(1,23 < z < 2,45) = P(-2,45 < z < -1,23)$

$= 0,4929 - 0,3907 = 0,1012$

(III) $P(-1,23 < z < 2,45) = ?$



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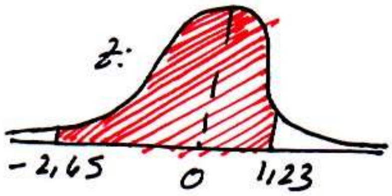
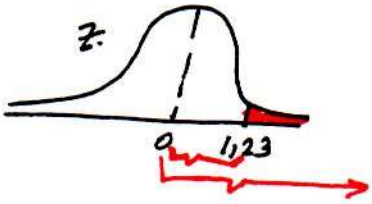


$P(-1,23 < z < 2,45) = 0,4929 + 0,3907 = 0,8836$

$= P(-2,45 < z < 1,23)$

(IV) (i) $P(z > 1,23) = ?$



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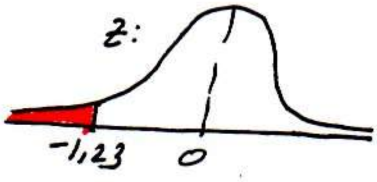
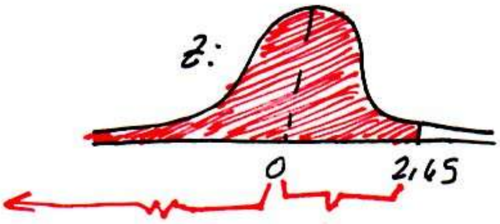
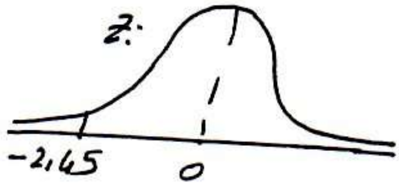


$P(z > 1,23) = 0,5 - 0,3907 = 0,1093 = P(z < -1,23)$

(ii) $P(Z < 2{,}45) = ?$

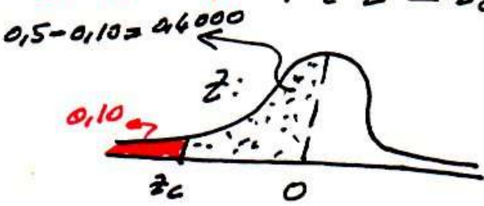


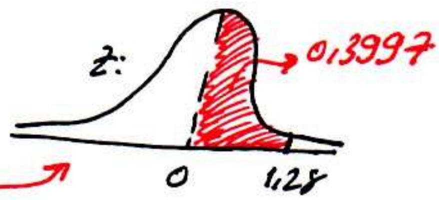
$=$



$P(Z < 2{,}45) = 0{,}5 + 0{,}4929 = 0{,}9929 = P(Z > -2{,}45)$

(V) (i) $P(Z < z_c) = 0{,}10 \Rightarrow z_c = ?$



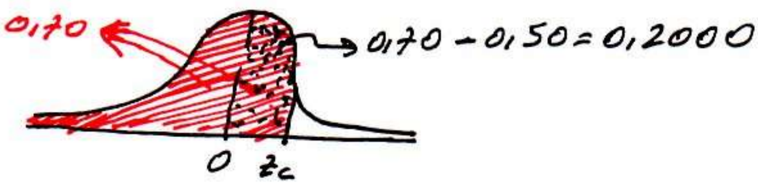


Nearest Number: $0{,}3997 \Rightarrow z_c = 1{,}28$

Negative Side $\Rightarrow \boxed{z_c = -1{,}28}$

(Note that, $P(Z > z_c) = 0{,}10 \Rightarrow z_c = 1{,}28$)

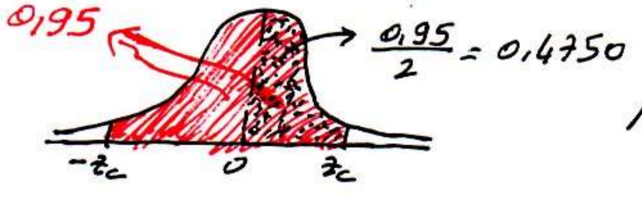
(ii) $P(Z < z_c) = 0{,}70 \Rightarrow z_c = ?$



Nearest Number: $0{,}1985 \Rightarrow z_c = 0{,}52$

Positive Side $\Rightarrow \boxed{z_c = 0{,}52}$

(iii) $P(-z_c < Z < z_c) = 0{,}95 \Rightarrow z_c = ?$



Nearest Number: $0{,}4750 \Rightarrow z_c = 1{,}96$

$\boxed{z_c = 1{,}96}$

* To find probabilities about $X \sim Normal(\mu ; \sigma^2)$ we convert X to Z by $Z = \frac{X-\mu}{\sigma}$

* To find numbers (or Number) X satisfying a probability, we find z_c and put in the equation

$$x_c = \mu + z_c \cdot \sigma$$

Example In a highschool, boys height has a normal distribution with mean 160 cm and variance 100. WPT a randomly selected boy has height

a) between 160 cm and 170 cm?

b) higher than 175 cm?

c) Less than 143 cm?

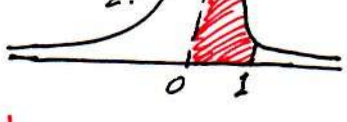
d) The heighesst 5% of the boys will be called to basketball team eliminations. What is the minimum height required to be called?

Answer $X \sim Normal(\mu = 160 ; \sigma^2 = 100)$

$$\sigma = \sqrt{100} = 10$$

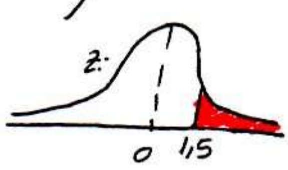
a) $P(160 < X < 170) = P\left(\frac{160-160}{10} < \underbrace{\frac{X-\mu}{\sigma}}_{=Z} < \frac{170-160}{10}\right)$

$= P(0 < Z < 1) = 0{,}3413$



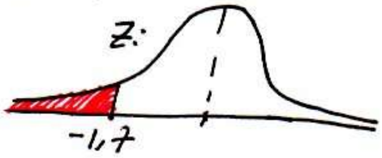
b) $P(X > 175) = P\left(\underbrace{\frac{X-\mu}{\sigma}}_{=Z} > \frac{175-160}{10}\right) = P(Z > 1{,}5)$

$= 0{,}5 - 0{,}4332 = 0{,}0668$

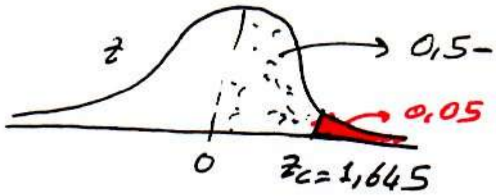


c) $P(X < 163) = P\left(\frac{X-\mu}{\sigma} < \frac{163-160}{10}\right) = P(Z < -1,7)$

$= 0,5 - 0,4554 = 0,0466$



d)



$0,5 - 0,05 = 0,4500$

	0,04	0.05
1,6	$\boxed{0.4495}$	$\boxed{0.4505}$

$Z = \frac{X-\mu}{\sigma}$

$1,645 = \frac{x_c - 160}{10} \Rightarrow x_c = 160 + 1,645 \cdot 10 = 176,45$

9.34 The reduction of a person's oxygen consumption during periods of transcendental meditation may be looked upon as a random variable having the normal distribution with $\mu = 38.6$ cc per minute and $\sigma = 6.5$ cc per minute. Find the probabilities that during a period of transcendental meditation a person's oxygen consumption will be reduced by

(a) at least 33.4 cc per minute;

(b) at most 34.7 cc per minute.

9.35 The sardines processed by a cannery have a mean length of 4.54 inches with a standard deviation of 0.25 inch. If the distribution of the lengths of the sardines can be approximated closely with a normal distribution, what percentage of the sardines are

(a) shorter than 4.00 inches;

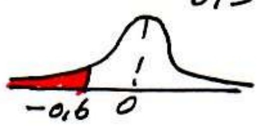
(b) between 4.40 and 4.60 inches long?

9.34) $X \sim Normal(\mu = 38,6 ; \sigma^2 = 6,5^2)$

a) $P(X \geq 33,4) = P\left(\frac{X-\mu}{\sigma} \geq \frac{33,4-38,6}{6,5}\right)$

$= P(Z \geq -0,8) = 0,5 + 0,2881 = 0,7881$

b) $P(X \leq 34,7) = P\left(Z \leq \frac{34,7-38,6}{6,5}\right) = P(Z \leq -0,6)$



$= 0,5 - 0,2257 = 0,2743$

9.35) $X \sim Normal(\mu = 4,54 ; \sigma^2 = 0,25^2)$

a) $P(X < 4) = P\left(Z < \frac{4-4,54}{0,25}\right) = P(Z < -2,16) = 0,5 - 0,4846$

$= 0,0154$

b) $P(4,40 < X < 4,60) = P\left(\frac{4,40-4,54}{0,25} < Z < \frac{4,60-4,54}{0,25}\right) = P(-0,56 < Z < 0,24)$

$= 0,2123 + 0,0948 = 0,3071$

Normal Approximation to Binomial Distribution

* For n is large and p is close to 0,5 (preferably $np(1-p)>9$), Normal Distribution provides a good approximation for Binomial Probabilities.

Let $X \sim \text{Binomial}(n; p)$

we set $\mu = n.p$ and $\sigma^2 = n.p.(1-p)$

and let $Y \sim \text{Normal}(\mu; \sigma^2)$

Then; $P(X \geq b) = P(Y > b - 0{,}5)$ and

$P(X \leq b) = P(Y < b + 0{,}5)$

We add or substract 0,5 because Binomial is a discrete distribution whereas Normal is continuous.

For example; $P(X=7) = P(6{,}5 < Y < 7{,}5)$

Remember that $P(Y=7)=0$ since Y is continuous.

Example A fair die is rolled 20 times. Use Normal approximation to obtain 12 heads.

Answer $X \sim \text{Binomial}(n=20; p=0{,}5)$

$\mu = n.p = 10 \; ; \; \sigma^2 = n.p.(1-p) = 5$

$Y \sim \text{Normal}(\mu=10; \sigma^2=5)$

$$P(X=12) = P(11{,}5 < Y < 12{,}5) = P\left(\frac{11{,}5-10}{\sqrt{5}} < Z < \frac{12{,}5-10}{\sqrt{5}}\right)$$

$$= P(0{,}67 < Z < 1{,}12) = 0{,}3686 - 0{,}2486 = 0{,}12$$

very close

Exact probability is; $P(X=12) = \binom{20}{12} 0{,}5^{12} \cdot 0{,}5^{8} = 0{,}120134$

Example. 17% of the graduate people get married with his/her flirt from University. If 300 people are graduated this year, WPT at most 65 of them will marry?

Answer $X \sim Binomial(n=300;\ p=0{,}17)$

$$\mu = 300 \cdot 0{,}17 = 51 \qquad \sigma^2 = 300 \cdot 0{,}17 \cdot 0{,}83 = 42{,}33$$

$$\sigma = \sqrt{42{,}33} = 6{,}51$$

$$Y \sim Normal(\mu = 51;\ \sigma^2 = 6{,}51^2)$$

$$P(X \leq 65) = P(Y < 65{,}5) = P\left(\frac{Y-\mu}{\sigma} < \frac{65{,}5-51}{6{,}51}\right)$$

$$= P(Z < 2{,}23) = 0{,}5 + 0{,}4871 = 0{,}9871$$

9.52 Use the normal distribution to approximate the probability that at least 55 of 90 persons flying across the Atlantic Ocean will feel the effect of the time difference for at least 24 hours, if the probability is 0.70 that any one person flying across the Atlantic Ocean will feel the effect of the time difference for at least 24 hours.

9.52) $X \sim Binomial(n=90;\ p=0{,}70)$

$$\mu = 90 \cdot 0{,}70 = 63;\quad \sigma^2 = 90 \cdot 0{,}70 \cdot 0{,}30 = 18{,}9$$

$$\sigma = \sqrt{18{,}9} = 4{,}35$$

$$Y \sim Normal(\mu = 63;\ \sigma^2 = 4{,}35^2)$$

$$P(X \geq 55) = P(Y > 54{,}5) = P\left(\frac{Y-\mu}{\sigma} > \frac{54{,}5-63}{4{,}35}\right)$$

$$= P(Z > -1{,}95) = 0{,}5 + 0{,}4744 = 0{,}9744$$