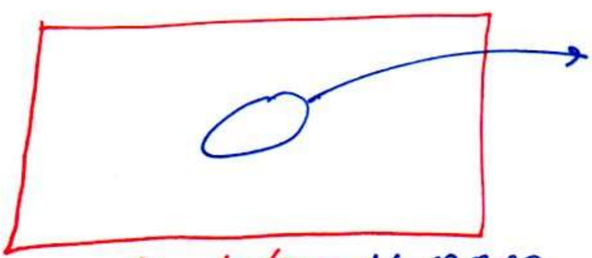
ECON STAT 2 LECTURE NOTES | CHAPTERS 10 & 11

HYPOTHESIS TESTING ⇒ YES/NO Questions

* Hypothesis testing questions are YES/NO questions
  "Can we conclude that ...?"
  "Is there sufficient evidence that ...?"
  "Is the claim true?" ... etc.

* We make inference about population parameters.



Random Sample $n=50$

Population $N=12000$

i.e. Bilkent University Students.

Ex: X_i: Weekly food expenditure of a student.
y_i: $=1$ if student smokes, 0 otherwise

	Population Parameters (Unknown Constants)	Sample Statistics (Known Variables)
MEAN	μ	$\bar{X} = \dfrac{\sum X_i}{n}$
VARIANCE	σ^2	$s^2 = \dfrac{\sum X_i^2 - \frac{(\sum X_i)^2}{n}}{n-1}$
PROPORTION	p	$\hat{p} = \dfrac{\sum y_i}{n}$

Basic Concepts;

* Hypothesis Testing steps;

(i) H_0, H_A and α

(ii) Test Statistics

(iii) Decision Criteria

(iv) Calculation

(v) Decision & Conclusion.

Example: I want to open a Restaurant at Bilkent. I think that opening the restaurant will be profitable if mean food expenditure of the students is more than 100 TL. Of a random sample of 50 students, mean food expenditure is found to be 103,76 TL. From past experience, variance is known to be 200.

a) At 5% significance level, should I open the restaurant?

H_0 and H_A are complementary.

Ans: (i) H_0, H_A and α

$H_0: \mu \leq 100$

$H_A: \mu > 100$ → inequality is always at H_A

$\alpha = 0{,}05$ α: Significance level OR Type-I Error

(ii) TEST STATISTICS

One sample mean test, σ^2 is known

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

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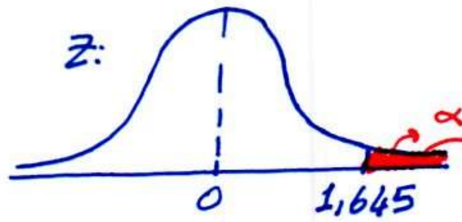
(iii) DECISION CRITERIA_1 (We have 3 forms)



→ Rejection Region, same side with H_A.

Reject H_0 if $Z > 1{,}645$

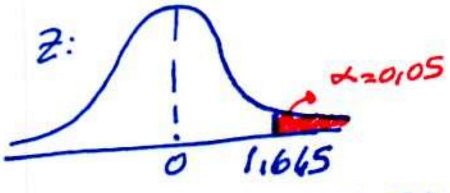
(The first form is decision criteria with respect to test statistics)

* Rejection Region is the same side with H_A.

Consider the following tests;

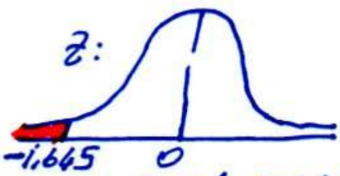
(This is called 2-sided test)

$H_0: \mu \leq 100$
$H_A: \mu > 100$
$\alpha = 0{,}05$



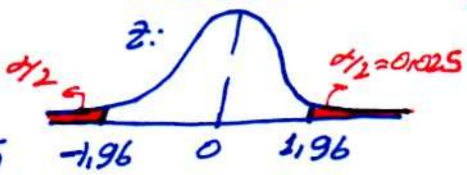
Reject H_0 if $Z > 1{,}645$

$H_0: \mu \geq 70$
$H_A: \mu < 70$
$\alpha = 0{,}05$



Reject H_0 if $Z < 1{,}645$

$H_0: \mu = 18$
$H_A: \mu \neq 18$
$\alpha = 0{,}05$



Reject H_0 if $|Z| > 1{,}96$

(iv) Calculation

$$\left.\begin{array}{l} n = 50 \\ \bar{X} = 103{,}76 \\ \sigma^2 = 200 \end{array}\right\} \Rightarrow Z = \frac{103{,}76 - 100}{\sqrt{200/50}} = 1{,}88$$

Decision & Conclusion

(v) 1,88 is in the rejection Region. Reject H_0. I can conclude that mean food expenditure is more than 100 TL and open the restaurant at $\alpha = 0{,}05$.

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b) At 1% significance level, should I open the restaurant?

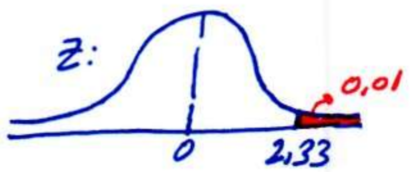
(i) $H_0: \mu \leq 100$

$H_A: \mu > 100$

$\alpha = 0{,}01$

(ii) $z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

(iii)



Reject H_0 if $z > 2{,}33$

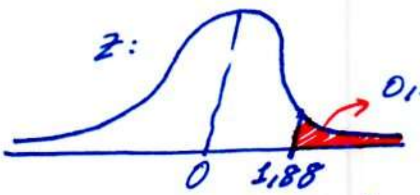
(iv) $z = \dfrac{103{,}76 - 100}{\sqrt{200/50}} = 1{,}88$

(v) Do NOT Reject H_0.
I can NOT conclude that mean food expenditure is more than 100 TL at $\alpha = 0{,}01$

* Note that my decision depends on significance level. If I can take 5% risk, I open the restaurant. However, If 5% is too much for me and max. risk I want to take is 1%, I do NOT open the restaurant.

c) What is the maximum risk of Type-I Error (Significance level) that I take if I want to open the restaurant?

$z = 1{,}88$



$0{,}5 - 0{,}4699 = 0{,}0301 \cong 3\%$

I Reject H_0 and open the restaurant for α values that are greater than 3%

This is called p-value!

p-value of the test

p-Value is the region (for 2-Sided test, $\frac{p\text{-value}}{2}$) of the test statistics that is with the same side of Alternative Hypothesis.

We Reject H_0 for α values that are greater than p-value. Or equivalently;

Decision Criteria-2 (in terms of p-value)

Reject H_0 if p-value $< \alpha$.

This decision criteria is the same for all sided tests.

Note that; p-value $= 0,03$

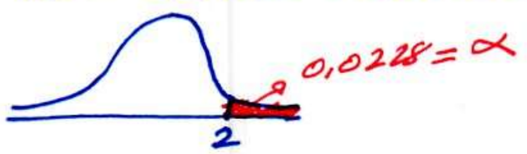
if $\alpha = 0,01$; p-value $> \alpha \Rightarrow$ Do NOT reject H_0.

if $\alpha = 0.05$; p-value $< \alpha \Rightarrow$ Reject H_0.

d) What is the significance level of the Decision Criteria "Reject H_0 if $\bar{X} > 104$"?

$$\alpha = P(\bar{X} > 104; \mu = 100) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{104-100}{\sqrt{200/50}}\right)$$

$$= P(Z > 2) = 0,5 - 0,4772 = 0,0228$$



$\bar{X} = 104 \Rightarrow Z = 2$

$\bar{X} > 104 \Rightarrow Z > 2$

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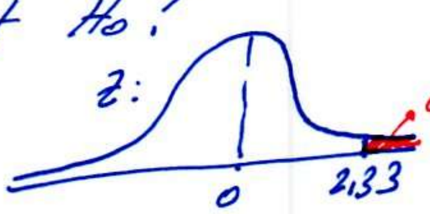
So; "Reject Ho if $\bar{X} > 104$" corresponds to the decision criteria "Reject Ho if $z > 2$" and also, equivalently "Reject Ho if p-value $< 0{,}0228$".

e) If $\alpha = 0{,}01$, what is the minimum sample mean to Reject Ho?



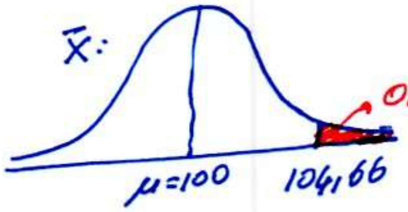
$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$2{,}33 = \frac{\bar{X}_c - 100}{\sqrt{200/50}}$$

$$\bar{X}_c = 104{,}66$$

Decision Criteria-3 (in terms of sample statistics)

Reject Ho if $\bar{X} > 104{,}66$



Type I and Type II Errors.

Ex: Ho: The person is innocent
HA: The person is convicted

True Case (Unknown)

Decision	Ho is true	HA is true
Do NOT Reject Ho	✓ 1-α confidence level	β: Type II Error
Reject Ho	α: Type I Error	✓ 1-β Power of the test.

$\alpha = P(\text{Reject Ho} ; Ho)$

$\beta = P(\text{Do NOT Reject Ho} ; H_A)$

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To find β-Error

(i) Write Decision Criteria - 3 (in terms of sample statistics)

(ii) Convert Decision Criteria-3 as "Do NOT Reject H_0 if..."

(iii) Find $\beta = P(\text{Do NOT Reject } H_0 \,;\, H_A)$

f) If true mean is 107 TL, find β-Error when $\alpha = 0{,}01$.

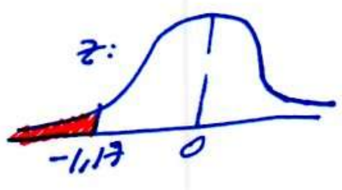
(i) Reject H_0 if $\bar{x} > 104{,}66$

(ii) Do NOT Reject H_0 if $\bar{x} \leq 104{,}66$

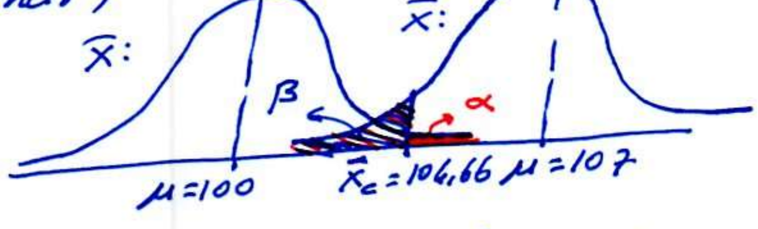
(iii) $\beta = P(\bar{x} \leq 104{,}66 ; \mu = 107) = P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \leq \frac{104{,}66 - 107}{\sqrt{200/50}}\right)$

$= P(z \leq -1{,}17) = 0{,}5 - 0{,}3790 = 0{,}121$

So; power of the test is $1 - 0{,}121 = 0{,}879$



* Note that;



* For a given sample size, if α decreases, β will increase and vice versa. To decrease both α and β, more sample should be collected or σ should be decreased if possible.

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HYPOTHESIS TESTS;

One Sample Tests
(Parameter vs. NUMBER)

MEAN

σ^2 known

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

σ^2 unknown

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}} \; ; \; \text{d.of } f = n-1$$

$$s^2 = \frac{\Sigma X_i^2 - (\Sigma X_i)^2 / n}{n-1} \; ; \; \bar{X} = \frac{\Sigma X_i}{n}$$

(For paired sample test, use t and replace $\bar{X}$ with $\bar{D}$; s with s_D)

VARIANCE

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

$$\text{d.of } f = n-1$$

PROPORTION

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$\hat{p} = \frac{X}{n}$; X: # of success

Two Sample tests
(Parameter_1 vs. Parameter_2)

MEAN

σ_i^2 are known

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

σ_i^2 are unknown, but assumed equal ($\sigma_1^2 = \sigma_2^2$)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2 / n_1 + s_p^2 / n_2}}$$

$$s_p^2 = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{\underbrace{n_1 + n_2 - 2}_{\text{d.of } f.}}$$

VARIANCE

$$F = \frac{s_1^2}{s_2^2}$$

larger variance $\leftarrow s_1^2 \rightarrow$ d.of $f = n_1 - 1$

$s_2^2 \rightarrow$ d.of $f = n_2 - 1$

PROPORTION

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2}$$

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10.12 A company that receives shipments of batteries tests a random sample of nine of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with standard deviation 3 hours. For one particular shipment the mean lifetime for a sample of nine batteries was 48.2 hours. Test at the 10% level the null hypothesis that the population mean lifetime is at least 50 hours.

10.12) $X \sim Normal(\mu; \sigma^2 = 3^2)$

$n = 9$; $\bar{X} = 48,2$; $\alpha = 0,10$

(i) $H_0: \mu \geq 50$

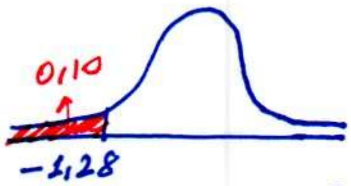
$H_A: \mu < 50$

$\alpha = 0,10$

(ii) One sample mean test, σ^2 known

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

(iii) Z:



Reject H_0 if $Z < -1,28$

(iv) $Z = \frac{48,2 - 50}{3/\sqrt{9}} = -1,8$

(v) Reject H_0. Mean is less than 50 at $\alpha = 0,10$.

10.13 A pharmaceutical manufacturer is concerned that the impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution with standard deviation 0.4%. A random sample of 64 pills from a production run was checked, and the sample mean impurity concentration was found to be 3.07%.

a. Test at the 5% level the null hypothesis that the population mean impurity concentration is 3% against the alternative that it is more than 3%.
b. Find the p-value for this test.
c. Suppose that the alternative hypothesis had been two-sided rather than one-sided (with null hypothesis H_0: $\mu = 3$). State, without doing the calculations, whether the p-value of the test would be higher than, lower than, or the same as that found in part (b). Sketch a graph to illustrate your reasoning.
d. In the context of this problem, explain why a one-sided alternative hypothesis is more appropriate than a two-sided alternative.

10.13 a)(i) $H_0: \mu \leq 3$

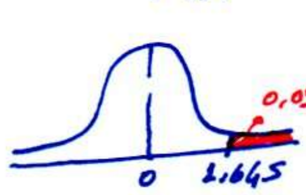
$H_A: \mu > 3$

$\alpha = 0,05$

(ii) One Sample Mean test, σ^2 known

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

(iii)



Reject H_0 if $Z > 1,645$

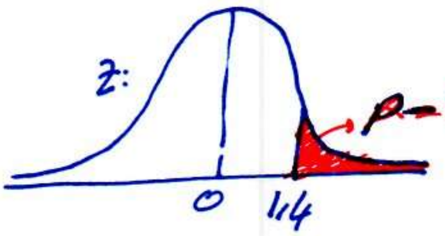
(iv) $\bar{X} = 3,07$; $\sigma = 0,4$

$Z = \frac{3,07 - 3}{0,4/\sqrt{64}} = 1,4$

(v) Do NOT Reject H_0. Mean is NOT greater than 3 at $\alpha = 0,05$

9

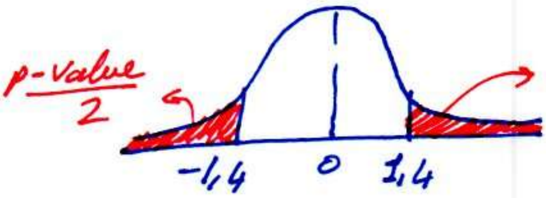
b)



p-value $= 0{,}5 - 0{,}4192 = 0{,}0808$

c) $H_0: \mu = 3$

$H_A: \mu \neq 3$



$\frac{p\text{-value}}{2} = 0{,}0808$

p-value $= 0{,}1616$

d) Because our concern is that if mean impurity exceeds 3%.

10.25 A statistics instructor is interested in the ability of students to assess the difficulty of a test they have taken. This test was taken by a large group of students, and the average score was 78.5. A random sample of eight students was asked to predict this average score. Their predictions were

72 83 78 65 69 77 81 71

Assuming a normal distribution, test the null hypothesis that the population mean prediction would be 78.5. Use a two-sided alternative and a 10% significance level.

10.25)

X_i	X_i^2
72	72^2
83	83^2
78	78^2
⋮	⋮
+ 71	71^2
$\Sigma X_i = 596$	$\Sigma X_i^2 = 44674$

$\bar{X} = \frac{596}{8} = 74{,}5$

$s^2 = \frac{44674 - 596^2/8}{7} = 38{,}86$

$\boxed{n = 8}$; d. of f = 7

(i) $H_0: \mu = 78{,}5$

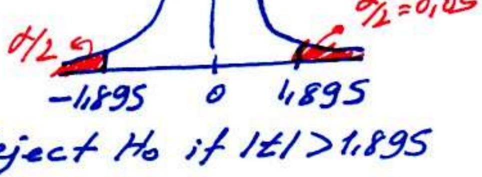
$H_A: \mu \neq 78{,}5$

$\alpha = 0{,}05$

(ii) One sample Mean test, σ^2 is unknown

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

(iii)



Reject H_0 if $|t| > 1{,}895$

(iv) $t = \frac{74{,}5 - 78{,}5}{\sqrt{38{,}86/8}} = 1{,}815$

(v) Do NOT reject H_0.

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10.27 In contract negotiations a company claims that a new incentive scheme has resulted in average weekly earnings of at least \$400 for all customer service workers. A union representative takes a random sample of 15 workers and finds that their weekly earnings have an average of \$381.35 and a standard deviation of \$48.60. Assume a normal distribution.

a. Test the company's claim.
b. If the same sample results had been obtained from a random sample of 50 employees, could the company's claim be rejected at a lower significance level than that used in part (a)?

a) $n = 15$; $\bar{X} = 381,35$; $s = 48,60$

(i) $H_0 : \mu \geq 400$

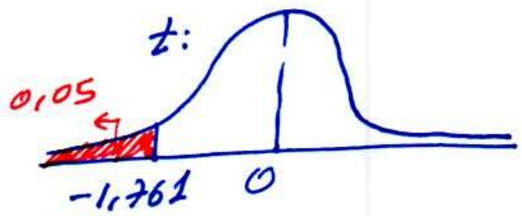
$H_A : \mu < 400$

$\alpha = 0,05$ → if NOT given.

(ii) One Sample Mean test, σ^2 unknown

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} \; ; \; d. \text{ of } f = 14$$

(iii)



Reject H_0 if $t < -1,761$

(iv) $t = \dfrac{381,35 - 400}{48,40/\sqrt{15}} = -1,49$

(v) Do NOT reject H_0.

10.31 In a random sample of 998 adults in the United States, 17.3% of the sample members indicated some measure of disagreement with this statement: "Globalization is more than an economic trade system—instead it includes institutions and culture." Test at the 5% level the hypothesis that at least 25% of all U.S. adults would disagree with this statement.

10.31) $n = 998$; $\hat{p} = 0,173$

(i) $H_0 : p \geq 0,25$

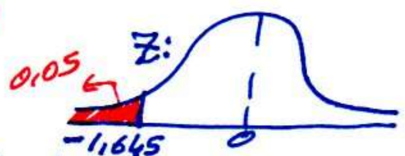
$H_A : p < 0,25$

$\alpha = 0,05$

(ii) One sample proportion test

$$Z = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$$

(iii)



Reject H_0 if $Z < -1,645$

(iv) $Z = \dfrac{0,173 - 0,25}{\sqrt{\dfrac{0,25 \cdot 0,75}{998}}} = -5,62$

(v) Reject H_0.

10.33 Of a random sample of 199 auditors, 104 indicated some measure of agreement with this statement: "Cash flow is an important indication of profitability." Test at the 10% significance level against a two-sided alternative the null hypothesis that one-half of the members of this population would agree with this statement. Also find and interpret the p-value of this test.

10.33) $n = 199$; $X = 104$; $\hat{p} = \frac{104}{199} = 0{,}523$

(i) $H_0: p = 0{,}5$

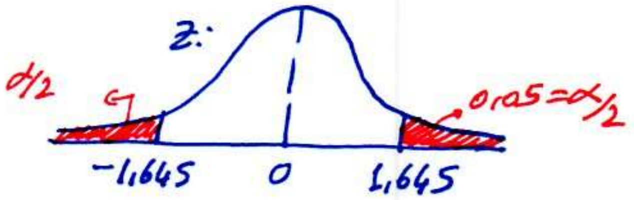
$H_A: p \neq 0{,}5$

$\alpha = 0{,}10$

(ii) One Sample Proportion test

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

(iii)



Reject H_0 if $|z| > 1{,}645$

(iv) $z = \frac{0{,}523 - 0{,}5}{\sqrt{\frac{0{,}5 \cdot 0{,}5}{199}}} = 0{,}65$

(v) Do NOT Reject H_0.

10.41 A random sample of 1,562 undergraduates enrolled in management ethics courses was asked to respond on a scale from 1 (strongly disagree) to 7 (strongly agree) to this proposition: "Senior corporate executives are interested in social justice." The sample mean response was 4.27, and the sample standard deviation was 1.32.

a. Test at the 1% level against a two-sided alternative the null hypothesis that the population mean is 4.
b. Find the probability of a 1%-level test accepting the null hypothesis when the true mean response is 3.95.

1.41) $n = 1562$; $\bar{x} = 4{,}27$; $s = 1{,}32$

a) (i) $H_0: \mu = 4$

$H_A: \mu \neq 4$

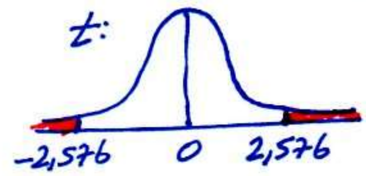
$\alpha = 0{,}01$

(ii) One sample Mean test, σ^2 unknown

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \text{ ; d.of f} = 1561$$

(Note that d.of f corresponds to ∞. we can also use Z-table)

(iii)



Reject H_0 if $|t| > 2{,}576$

(iv) $t = \frac{4{,}27 - 4}{1{,}32/\sqrt{1562}} = 8{,}08$ (v) Reject H_0.

b) $Z = 2,576$; $Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$ $\quad$ $Z = -2,576$

$$2,576 = \frac{\bar{X}_c - 4}{1,32/\sqrt{1562}} \qquad -2,576 = \frac{\bar{X}_c - 4}{2,32/\sqrt{1561}}$$

$$\bar{X}_c = 4,086 \qquad \bar{X}_c = 3,914$$

Reject Ho if $\bar{X} > 4,086$ OR $\bar{X} < 3,914$

Do NOT reject Ho if $3,914 < \bar{X} < 4,086$

$$\beta = P(3,914 < \bar{X} < 4,086;\ \mu = 3,95) = P\left(\frac{3,914 - 3,95}{1,32/\sqrt{1562}} < Z < \frac{4,086 - 3,95}{1,32/\sqrt{1562}}\right)$$

$$= P(-1,06 < Z < 4,07) = 0,5 + 0,3554 = \underline{0,8554}$$

β-Error is High because $\mu = 3,95$ is too close to 4.

10.42 A random sample of 802 supermarket shoppers had 378 shoppers that preferred generic brand items if the price was lower. Test at the 10% level the null hypothesis that at least one-half of all shoppers preferred generic brand items against the alternative that the population proportion is less than one-half. Find the power of a 10%-level test if, in fact, 45% of the supermarket shoppers are able to state the correct price of an item immediately after putting it into the cart.

10.42) $n = 802$; $X = 378$; $\hat{p} = \frac{378}{802} = 0,471$

(i) $H_0 : p \geq 0,5$

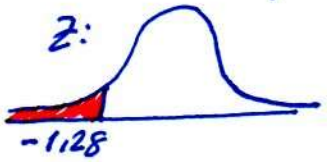
$H_A : p < 0,5$

$\alpha = 0,10$

(ii) One sample proportion test

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

(iii) Z:



Reject Ho if $Z < -1,28$

(iv) $Z = \dfrac{0,471 - 0,5}{\sqrt{\frac{0,5 \cdot 0,5}{802}}} = -1,64$

(v) Reject Ho.

paired sample t-test

Ex ⇒ ~~The~~ salesperson had a motivation course and their sales before and after the course is recorded. Is the course increased mean sales?

Salesperson	Before The Course	After The Course	D_i: Difference (After − Before)	D_i^2
1.	212	237	25	25^2
2.	282	291	9	9^2
3.	203	191	−12	$(-12)^2$
4.	327	341	14	14^2
5.	165	192	27	27^2
6.	198	180	+ −18	+ $(-18)^2$
			$\Sigma D_i = 45$	$\Sigma D_i^2 = 2099$

$\boxed{n=6}$

$\bar{D} = \frac{45}{6} = 7{,}5 \qquad S_D^2 = \frac{2099 - 45^2/6}{5} = 352{,}3$

$\mu_D = \mu_A - \mu_B$. If Sales are increased, $\mu_D > 0$

(i) $H_0: \mu_D \le 0$

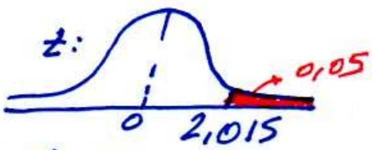
$H_A: \mu_D > 0$

$\alpha = 0{,}05$

(ii) Paired sample test

$t = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}}$; d.off = 5

(iii)



Reject H_0 if $t > 2{,}015$

(iv) $t = \frac{7{,}5 - 0}{\sqrt{352{,}3/6}} = 0{,}98$

(v) Do NOT Reject H_0.
Motivation Course is NOT efficient at $\alpha = 0{,}05$

11.3 In a study comparing banks in Germany and Great Britain, a sample of 145 matched pairs of banks was formed. Each pair contained one bank from Germany and one from Great Britain. The pairings were made in such a way that the two members were as similar as possible in regard to such factors as size and age. The ratio of total loans outstanding to total assets was calculated for each of the banks. For this ratio, the sample mean difference (German − Great Britain) was 0.0518, and the sample standard deviation of the differences was 0.3055. Test against a two-sided alternative the null hypothesis that the two population means are equal.

11.3) $n = 145;\ \bar{D} = 0{,}0518;\ s_D = 0{,}3055$

(i) $H_0: \mu_D = 0$

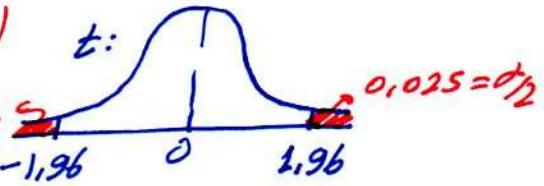
$H_A: \mu_D \neq 0$

$\alpha = 0{,}05$

(ii) Paired Sample t-test

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} \quad \text{d.o.f} = 144$$

(iii)



Reject H_0 if $|t| > 1{,}96$

(iv) $$t = \frac{0{,}0518 - 0}{0{,}3055/\sqrt{145}} = 2{,}04$$

(v) Reject H_0.

11.4 A screening procedure was designed to measure attitudes toward minorities as managers. High scores indicate negative attitudes and low scores indicate positive attitudes. Independent random samples were taken of 151 male financial analysts and 108 female financial analysts. For the former group the sample mean and standard deviation scores were 85.8 and 19.13, while the corresponding statistics for the latter group were 71.5 and 12.2. Test the null hypothesis that the two population means are equal against the alternative that the true mean score is higher for male than for female financial analysts.

11.4)

Male	Female
$n_1 = 151$	$n_2 = 108$
$\bar{X}_1 = 85{,}8$	$\bar{X}_2 = 71{,}5$
$s_1 = 19{,}13$	$s_2 = 12{,}2$

Large samples $n_1, n_2 > 30$ we may assume σ_i^2 known (Replace with s_i^2)

(i) $H_0: \mu_1 = \mu_2$

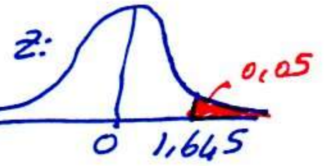
$H_A: \mu_1 > \mu_2$

$\alpha = 0{,}05$

(ii) Two sample Mean test, σ_i^2 known.

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

(iii)



Reject H_0 if $z > 1{,}645$

(iv) $$z = \frac{85{,}8 - 71{,}5}{\sqrt{\frac{19{,}13^2}{151} + \frac{12{,}2^2}{108}}} = 7{,}334$$

(v) Reject H_0.

11.9 A publisher is interested in the effects on sales of college texts that include more than 100 data files. The publisher plans to produce 20 texts in the business area and randomly chooses 10 to have more than 100 data files. The remaining 10 are produced with at most 100 data files. For those with more than 100, first-year sales averaged 9,254, and the sample standard deviation was 2,107. For the books with at most 100, average first-year sales were 8,167, and the sample standard deviation was 1,681. Assuming that the two population distributions are normal with the same variance, test the null hypothesis that the population means are equal against the alternative that the true mean is higher for books with more than 100 data files.

11.9)

More than 100	At Most 100
$n_1 = 10$	$n_2 = 10$
$\bar{X}_1 = 9254$	$\bar{X}_2 = 8167$
$s_1 = 2107$	$s_2 = 1681$

$X_1 \sim Normal(\mu_1, \sigma^2)$

$X_2 \sim Normal(\mu_2, \sigma^2)$

(i) $H_0: \mu_1 \leq \mu_2$

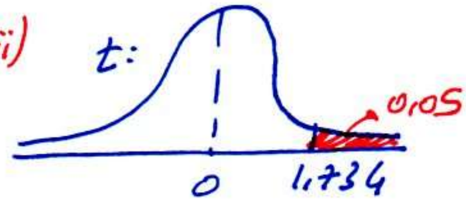
$H_A: \mu_1 > \mu_2$

$\alpha = 0{,}05$

(ii) Two Sample Mean test, σ_i^2 unkown, $\sigma_1^2 = \sigma_2^2$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} \; ; d.of\,f = 18$$

(iii) t:



Reject H_0 if $t > 1{,}734$

(iv) $s_p^2 = \frac{9.2107^2 + 9.1681^2}{18} = 1894^2$

$$t = \frac{9254 - 8167}{\sqrt{1894^2\left(\frac{1}{10} + \frac{1}{10}\right)}} = 1{,}32$$

(v) Do NOT Reject H_0.

11.14 A random sample of 1,556 people in country A were asked to respond to this statement: "Increased world trade can increase our per capita prosperity." Of these sample members, 38.4% agreed with the statement. When the same statement was presented to a random sample of 1,108 people in country B, 52.0% agreed. Test the null hypothesis that the population proportions agreeing with this statement were the same in the two countries against the alternative that a higher proportion agreed in country B.

11.14)

Country A	Country B
$n_A = 1556$	$n_B = 1108$
$\hat{p}_A = 0{,}384$	$\hat{p}_B = 0{,}52$

(i) $H_0: p_A \geq p_B$

$H_A: p_A < p_B$

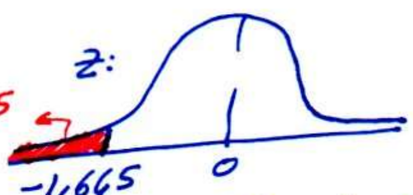
$\alpha = 0{,}05$

$$\hat{p} = \frac{0{,}384.1556 + 0{,}52.1108}{1556 + 1108} = 0{,}441$$

(ii) Two sample proportion test

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

(iii)



Reject Ho if $z < -1,645$

(iv) $$z = \frac{0,384 - 0,52}{\sqrt{0,441 \cdot (1-0,441) \cdot \left(\frac{1}{1556} + \frac{1}{1108}\right)}} = -6,97$$

(v) Reject Ho.

11.18 Independent random samples of consumers were asked about satisfaction with their computer system in two slightly different ways. The options available for answer were the same in the two cases. When asked how *satisfied* they were with their computer system, 138 of 240 sample members opted for "very satisfied." When asked how *dissatisfied* they were with their computer system, 128 of 240 sample members opted for "very satisfied." Test at the 5% significance level, against the obvious one-sided alternative, the null hypothesis that the two population proportions are equal.

11.18)

Satisfied	Dissatisfied
$X_1 = 138$	$X_2 = 128$
$n_1 = 240$	$n_2 = 240$
$\hat{p}_1 = \frac{138}{240} = 0,575$	$\hat{p}_2 = \frac{128}{240} = 0,533$

$$\hat{p} = \frac{138+128}{240+240} = 0,554$$

(i) $H_0: p_1 \le p_2$

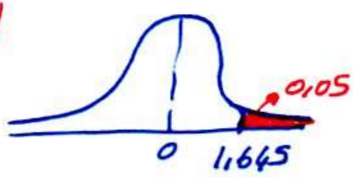
$H_A: p_1 > p_2$

$\alpha = 0,05$

(ii) Two sample proportion test

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

(iii)



Reject Ho if $z > 1,645$

(iv) $$z = \frac{0,575 - 0,533}{\sqrt{0,554 \cdot (1-0,554)\left(\frac{1}{240}+\frac{1}{240}\right)}} = 0,926$$

(v) Do NOT reject Ho.

11.21 At the insistence of a government inspector a new safety device is installed in an assembly-line operation. After the installation of this device a random sample of 8 days' output gave the following results for numbers of finished components produced:

618 660 638 625 571 598 639 582

Management is concerned about the variability of daily output and views as undesirable any variance above 500. Test at the 10% significance level the null hypothesis that the population variance for daily output does not exceed 500.

11.21)

X_i	X_i^2
618	618^2
660	660^2
638	638^2
625	625^2
571	571^2
598	598^2
639	639^2
582	582^2
$\sum X_i = 4931$	$\sum X_i^2 = 3045883$

$\boxed{n=8}$

$$\bar{X} = \frac{4931}{8} = 616,4 \qquad s^2 = \frac{3045883 - 4931^2/8}{7} = 934$$

(i) $H_0: \sigma^2 \leq 500$

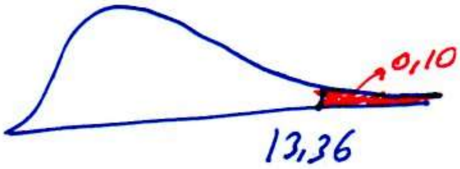
$H_A: \sigma^2 > 500$

$\alpha = 0,10$

(ii) One sample Variance Test

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad d.of f = 7$$

(iii)



Reject H_0 if $\chi^2 > 13,36$

(iv) $$\chi^2 = \frac{7 \cdot 934}{500} = 13,08$$

Do NOT Reject H_0.

11.23 One way to evaluate the effectiveness of a teaching assistant is to examine the scores achieved by his or her students on an examination at the end of the course. Obviously, the mean score is of interest. However, the variance also contains useful information—some teachers have a style that works very well with more able students but is unsuccessful with less able or poorly motivated students. A professor sets a standard examination at the end of each semester for all sections of a course. The variance of the scores on this test is typically very close to 300. A new teaching assistant has a class of 30 students, whose test scores had a variance of 480. Regarding these students' test scores as a random sample from a normal population, test against a two-sided alternative the null hypothesis that the population variance of their scores is 300.

11.23) $n = 30;\ s^2 = 480$

(i) $H_0: \sigma^2 = 300$

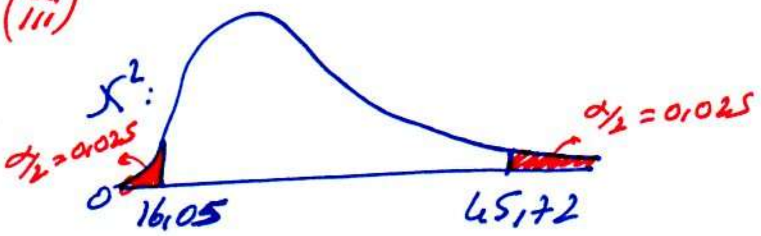
$H_A: \sigma^2 \neq 300$

$\alpha = 0{,}05$

(ii) One sample Variance Test

$$\chi^2 = \frac{(n-1).\, s^2}{\sigma^2} \quad \text{d.of f} = 29$$

(iii)



Reject H_0 if $\chi^2 \leq 16{,}75$

OR $\chi^2 \geq 45{,}72$

(iv) $\chi^2 = \frac{29.480}{300} = 46{,}4$

(v) Reject H_0.

11.30 In Exercise 11.9 it was assumed that population variances were equal for first-year sales of textbooks with more than 100 data files and those with at most 100 data files. Test this assumption against a two-sided alternative.

11.30)

More than 100	At Most 100
$n_1 = 10$	$n_2 = 10$
$s_1 = 2107$	$s_2 = 1681$

(i) $H_0: \sigma_1^2 = \sigma_2^2$

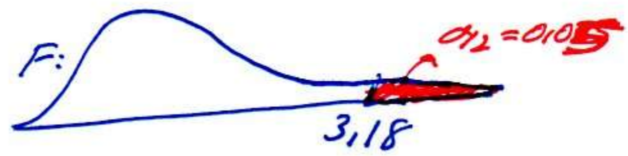
$H_A: \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0{,}10$

(ii) Two Sample Variance Test

$$F = \frac{s_1^2 \rightarrow \text{d.of f} = 9}{s_2^2 \rightarrow \text{d.of f} = 9}$$

(iii)



Reject H_0 if $F > 3{,}18$

(iv) $F = \frac{2107^2}{1681^2} = 1{,}57$

(v) Do Not Reject H_0. We can assume Equal Variances.

11.31 A university research team was studying the relationship between idea generation by groups with and without a moderator. For a random sample of four groups with a moderator the mean number of ideas generated per group was 78.0, and the standard deviation was 24.4. For a random sample of four groups without a moderator the mean number of ideas generated was 63.5, and the standard deviation was 20.2 Test the assumption that the two population variances were equal against the alternative that the population variance is higher for groups with a moderator.

11.31)

with Mod.	Without Mod.
$n_1 = 4$	$n_2 = 4$
$\bar{X}_1 = 78$	$\bar{X}_2 = 63,5$
$s_1 = 24,4$	$s_2 = 20,2$

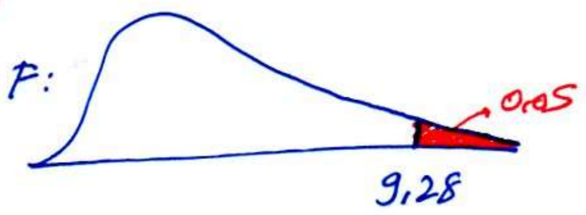
(i) $H_0: \sigma_1^2 \leq \sigma_2^2$

$H_A: \sigma_1^2 > \sigma_2^2$

$\alpha = 0,05$

(ii) Two Sample Proportion Test

$$F = \frac{s_1^2 \rightarrow \text{d.of f}=3}{s_2^2 \rightarrow \text{d.of f}=3}$$

(iii) 

Reject H_0 if $F > 9,28$

(iv) $F = \frac{24,4^2}{20,2^2} = 1,45$

(v) Do NOT Reject H_0.