



ECON STAT-2 LECTURE NOTES	CHAPTERS 16 & 17
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## GOODNESS-OF-FIT TESTS

### (I) Specified Probabilities

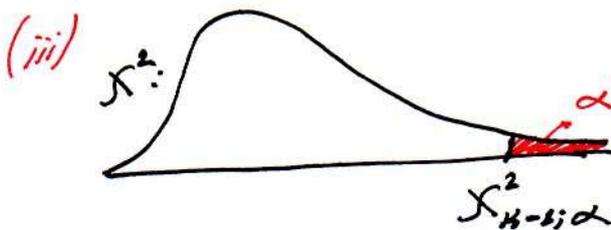
\* We test if the specified probabilities fits to observed data. We have the following test, in general;

(i)  $H_0: p^1 = p_1; p^2 = p_2; \dots; p^k = p_k$

$H_A$ : At least one probability is different

where,  $\sum_{i=1}^k p_i = 1$

(ii)  $\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$ ; deg. of freedom =  $k - 1$   
 (since no parameter is estimated from the sample data)



Reject  $H_0$  if  $\chi^2 > \chi^2_{k-1, \alpha}$

(iv) Remember from Binomial distribution;  $E(X) = n \cdot p$ .

Likewise, here we have  $E_i = n \cdot p_i$   
 If "equally likely" or "evenly distributed";  $p_i = \frac{1}{k}$ .

16.3 An insurance company in Chattanooga, Tennessee, wanted to determine the importance of price as a factor in choosing a hospital in that region. A random sample of 450 consumers was asked to select "not important," "important," or "very important" as an answer. Respective numbers selecting these answers were 142, 175, and 133. Test the null hypothesis that a randomly chosen consumer is equally likely to select each of these three answers.

16.3)  $k = 3$  categories,  
 "Equally Likely"  $\Rightarrow p_i = \frac{1}{3}; i = 1, 2, 3$

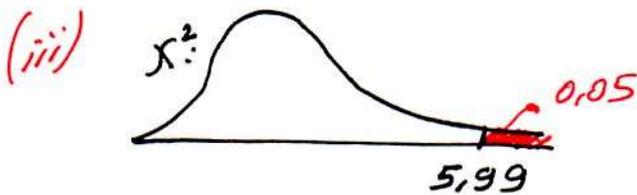
Note that: This is "discrete Uniform Distribution"

(i)  $H_0: p_1 = p_2 = p_3 = \frac{1}{3}$  (Price factor's importance have Uniform distribution)

$H_A$ : At least one  $p_i \neq \frac{1}{3}$  (The distribution is NOT Uniform)

$\alpha = 0,05$

(ii)  $\chi^2 = \sum_{i=1}^3 \frac{(O_i - E_i)^2}{E_i}; d.o.f = 3 - 1 = 2$



Reject  $H_0$  if  $\chi^2 > 5,99$

(iv)  $n = 450$

Importance Category	(1) "Not important"	(2) "Important"	(3) "Very Important"
$O_i$	142	175	133
$p_i$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$E_i$	$450 \cdot \frac{1}{3} = 150$	$450 \cdot \frac{1}{3} = 150$	$450 \cdot \frac{1}{3} = 150$

$\chi^2 = \frac{(142 - 150)^2}{150} + \frac{(175 - 150)^2}{150} + \frac{(133 - 150)^2}{150} = 6,52$

(v) Reject  $H_0$ . The distribution is NOT uniform at  $\alpha = 0,05$  61

16.9 A team of marketing research students was asked to determine the pizza best liked by students enrolled in their college. Two years ago a similar study was conducted, and it was found that 40% of all students at this college preferred Bellini's pizza, 25% chose Anthony's pizza as the best, 20% selected Ferrara's pizza, and the rest selected Marie's pizza. To see if preferences have changed, 180 students were randomly selected and asked to indicate their pizza preferences. The results were as follows: 40 selected Ferrara's as their favorite, 32 students chose Marie's, 80 students preferred Bellini's, and the remainder selected Anthony's. Do the data indicate that the preferences today differ from those from the last study?

16.9)  $K = 4$  Categories;  $n = 180$

Pizza:	(1) Bellini	(2) Anthony	(3) Ferrara	(4) Marie
$O_i$	80	$\frac{180 - 80 - 40 - 32}{1} = 28$	40	32
$P_i$	0.40	0.25	0.20	$1 - 0.40 - 0.25 - 0.20 = 0.15$
$E_i$	$180 \cdot 0.40 = 72$	$180 \cdot 0.25 = 45$	$180 \cdot 0.20 = 36$	$180 \cdot 0.15 = 27$

(i)  $H_0: p_1 = 0.40; p_2 = 0.25; p_3 = 0.20; p_4 = 0.15$

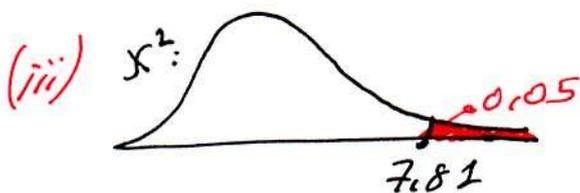
(Preferences of students is the same as 2 years ago)

$H_A$ : At least one  $p_i$  is different

(Preferences have changed)

$\alpha = 0.05$

(ii)  $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$ ; d.o.f =  $4 - 1 = 3$



Reject  $H_0$  if  $\chi^2 > 7.81$

(iv)  $\chi^2 = \frac{(80 - 72)^2}{72} + \frac{(28 - 45)^2}{45} + \frac{(40 - 36)^2}{36} + \frac{(32 - 27)^2}{27} = 8.68$

(v) Reject  $H_0$ . Preferences have changed at  $\alpha = 0.05$

Ex: what is the p-value of the test?

Note that  $7.81 = \chi_{0.05; 3}^2 < 8.68 < \chi_{0.025; 3}^2 = 9.35$

Then;  $0.05 < p\text{-value} < 0.025$ ; Do Not Reject  $H_0$  at  $\alpha = 0.025$  or lower (62)



## (II) Population Parameters Unknown

\* We will see Poisson-fit and Normal-fit tests.

Remember;  $X \sim \text{Poisson}(\lambda)$

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}; \quad \lambda = E(X)$$

If we estimate  $\lambda$  as  $\hat{\lambda} = \frac{\sum \lambda_i}{\sum t_i}$ ; we lose one

degrees of freedom; then, d.o.f =  $K - 1 - 1 = K - 2$ .

Remember; d.o.f =  ~~$n - 1$~~  - # of estimated parameters from sample data. (For example, Multiple Regression)

$$\text{Then; } P_x = \frac{e^{-\hat{\lambda}} \cdot \hat{\lambda}^x}{x!} \text{ and } E_x = n \cdot P_x$$

\* For Normal-fit case, we will learn Bowman-Shelton statistics where;

$$B = n \cdot \left[ \frac{(\text{skewness})^2}{6} + \frac{(\text{Kurtosis} - 3)^2}{24} \right]$$

This test follows because of normal distribution has Skewness = 0 and Kurtosis = 3. For a perfect Normal distribution,  $B = 0$ . The test allows up to some levels, departures from Normality via skewness and Kurtosis. The significance points for Bowman-Shelton statistics for specific  $n$  is given in the following table. Note that test approaches to  $\chi^2(2)$  at infinity.

Sample size:n	10% point	5% point	Sample size:n	10% point	5% point
20	2.13	3.26	200	3.48	4.43
30	2.49	3.71	250	3.54	4.51
40	2.70	3.99	300	3.68	4.60
50	2.90	4.26	400	3.76	4.74
75	3.09	4.27	500	3.91	4.82
100	3.14	4.29	800	4.32	5.46
125	3.31	4.34	$\infty$	4.61	5.99
150	3.43	4.39			

16.14 A random sample of 100 measurements of the resistance of electronic components produced in a period of 1 week was taken. The sample skewness was 0.63 and the sample kurtosis was 3.85. Test the null hypothesis that the population distribution is normal.

16.14)  
 $n = 100$ ; skewness = 0.63

Kurtosis = 3.85

(i)  $H_0$ : Population distribution is Normal  
 $H_A$ : Data does NOT follow a Normal distribution

$\alpha = 0.05$

(ii) 
$$B = n \cdot \left[ \frac{(\text{skewness})^2}{6} + \frac{(\text{Kurtosis} - 3)^2}{24} \right]$$

(iii) Reject  $H_0$  if  $B > 4.29$  (Like  $\chi^2$ )

(iv) 
$$B = 100 \cdot \left[ \frac{0.63^2}{6} + \frac{(3.85 - 3)^2}{24} \right] = 9.63$$

(v) Reject  $H_0$ . Data does NOT follow a Normal distribution at  $\alpha = 0.05$ .

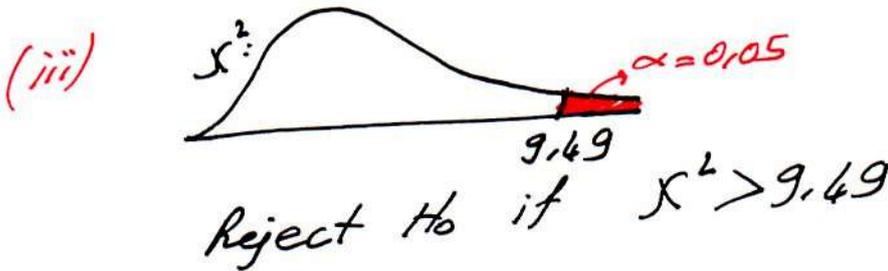
16.11 The number of times a machine broke down each week was observed over a period of 100 weeks and recorded in the accompanying table. It was found that the average number of breakdowns per week over this period was 2.1. Test the null hypothesis that the population distribution of breakdown is Poisson.

Number of Breakdowns	0	1	2	3	4	5 or More
Number of weeks	10	24	32	23	6	5

(Data is given in the question)

- (i)  $H_0$ : The population distribution is Poisson  
 $H_A$ : Data does NOT follow a Poisson distribution.  
 $\alpha = 0,05$ .

(ii)  $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$ ; d.o.f =  $k - 2 = 6 - 2 = 4$



(iv)  $\hat{\lambda} = 2,1$ ;  $X$ : # of Breakdowns / week ;  $n = 100$

$X \sim \text{Poisson}(\hat{\lambda} = 2,1)$  under  $H_0$ .

$f(x) = \frac{e^{-2,1} \cdot 2,1^x}{x!}$ ;  $P(X=0) = p_0 = \frac{e^{-2,1} \cdot 2,1^0}{0!} = 0,122$

$P(X=1) = p_1 = \frac{e^{-2,1} \cdot 2,1^1}{1!} = 0,257$   
 $\vdots$

$P(X \geq 5) = p_5 = 1 - p_0 - p_1 - p_2 - p_3 - p_4$

# of breakdowns	0	1	2	3	4	5 or More
$O_i$	10	24	32	23	6	5
$P_i$	0,122	0,257	0,269	0,188	0,099	0,065
$E_i = P_i \cdot 100$	12,2	25,7	26,9	18,8	9,9	6,5

$$\chi^2 = \frac{(10-12,2)^2}{12,2} + \frac{(26-25,7)^2}{25,7} + \dots + \frac{(5-6,5)^2}{6,5}$$

$$\chi^2 = 4,297$$

(v) Do NOT Reject  $H_0$ . Poisson distribution can be assumed at  $\alpha = 0,05$ . (Note that we do NOT reject  $H_0$  at  $\alpha = 0,10$  also. Check this!)

Note: In q. 16.11, it can also be stated that "Over the 100 week period, the total # of breakdowns were 210". Then;  $\hat{\lambda} = \frac{210}{100} = 2,1$  would be calculated.

## CONTINGENCY TABLES

\* Contingency table is a frequency table representing frequencies of two categorical variables.

Row Variable	1	2	...	j	...	c	Total
1	$O_{11}$	$O_{12}$	...	$O_{1j}$	...	$O_{1c}$	$T_{1.}$
2	$O_{21}$	$O_{22}$	...	$O_{2j}$	...	$O_{2c}$	$T_{2.}$
...	...	...	...	$O_{ij}$	...	$O_{ic}$	$T_{i.}$
r	$O_{r1}$	$O_{r2}$	...	$O_{rj}$	...	$O_{rc}$	$T_{r.}$
Total	$T_{.1}$	$T_{.2}$	...	$T_{.j}$	...	$T_{.c}$	$T_{..}$

We have:  $E_{ij} = \frac{T_{i.} \times T_{.j}}{T_{..}}$

r: # of rows  
c: # of columns.

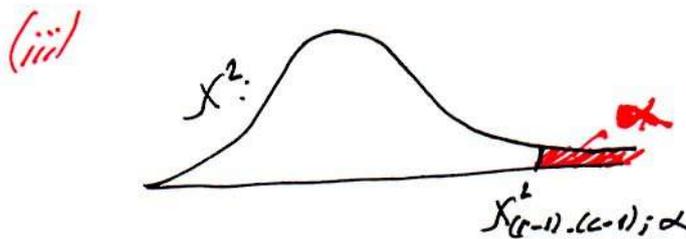
$$\chi^2 = \sum \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \text{ with d.o.f } (r-1) \cdot (c-1)$$



\* Here, what we test is the association between row variable and Column Variable. If row variable and column variable is independent, the Observed values will be close to Expected Values and  $\chi^2$  value will be low, supporting  $H_0$ : independence. However, if Observed values aim to change via specific levels, there will be differences between  $O_{ij}$  and  $E_{ij}$ , resulting in high  $\chi^2$  value supporting  $H_A$ : association (dependence)

(i)  $H_0$ : Column Variable and Row Variable is independent  
 $H_A$ : There exist a relationship between them.

(ii) 
$$\chi^2 = \sum \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}; \text{ d.o.f} = (r-1) \cdot (c-1)$$



Reject  $H_0$  if  $\chi^2 > \chi^2_{(r-1) \cdot (c-1); \alpha}$

(iv) 
$$E_{ij} = \frac{T_{i.} \times T_{.j}}{T_{..}}$$
 where  $T_{i.}$ : Row total of row  $i$   
 $T_{.j}$ : column total of column  $j$   
 $T_{..}$ : Grand Total

Calculate  $E_{ij}$  for each cell in the table and calculate  $\chi^2$

(v) Decide & Conclude

16.18 University administrators have collected the following information concerning student grade point average and the school of the student's major.

School	GPA < 3.0	GPA 3.0 or Higher
Arts & Sciences	50	35
Business	45	30
Music	15	25

Determine if there is any association between GPA and major.

6-18)

School	GPA < 3.0	GPA ≥ 3.0	TOTAL
Arts & Sciences	50 (66,75)	35 (38,25)	85
Business	45 (61,25)	30 (33,75)	75
Music	15 (22)	25 (18)	40
TOTAL	110	90	200

$$E_{11} = \frac{85 \cdot 110}{200} = 66,75 ; E_{21} = \frac{75 \cdot 110}{200} = 61,25 \quad r=3; c=2$$

$$E_{12} = \frac{85 \cdot 90}{200} = 85 - 66,75 = 38,25 ; E_{22} = \frac{75 \cdot 90}{200} = 75 - 61,25 = 33,75$$

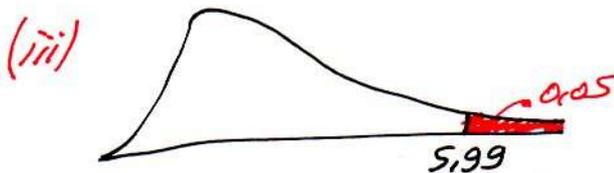
$$E_{13} = 110 - (66,75 + 61,25) = 22 \quad E_{23} = 40 - 22 = 18$$

(i)  $H_0$ : School and being Honour student is independent

$H_A$ : There exist a relationship between them

$$\alpha = 0,05$$

(ii)  $\chi^2 = \sum \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} ; d.o.f = (3-1) \cdot (2-1) = 2$



Reject  $H_0$  if  $\chi^2 > 5,99$

(iv)  $\chi^2 = \frac{(50 - 66,75)^2}{66,75} + \frac{(35 - 38,25)^2}{38,25} + \dots + \frac{(25 - 18)^2}{18} = 6,21$

(v) Reject  $H_0$ . Honour students change with respect to school (There's an association) at  $\alpha = 0,05$

16.29 A manufacturer of household appliances wanted to determine if there was a relationship between family size and the size of washing machine purchased. The manufacturer was preparing guidelines for sales personnel and wanted to know if the sales staff should make specific recommendations to customers. A random sample of 300 families was asked about family size and size of washing machine. For the 40 families with one or two people, 25 had an 8-pound washer, 10 had a 10-pound washer, and 5 had a 12-pound washer. The 140 families with three or four people included 37 with the 8-pound, 62 with the 10-pound, and 41 with the 12-pound. For the remaining 120 families with five or more people, 8 had an 8-pound, 53 had a 10-pound, and 59 had a 12-pound. Based on these results, what can be concluded about family size and size of washer? Construct a two-way table, state the hypothesis, compute the statistics, and state your conclusion.

6.29)  $n = 300$

Family Size	Size of Washing Mach.			TOTAL
	8-pound	10-pound	12-pound	
1-2	25 (9,3)	10 (16,7)	5 (14)	40
3-4	37 (32,7)	62 (58,3)	41 (49)	140
5 or More	8 (28)	53 (50)	59 (42)	120
TOTAL	70	125	105	300

$$E_{11} = \frac{40 \cdot 70}{300} = 9,3 \quad E_{12} = \frac{40 \cdot 125}{300} = 16,7 \quad E_{13} = 40 - 9,3 - 16,7 = 14$$

$$E_{21} = \frac{140 \cdot 70}{300} = 32,7 \quad E_{22} = \frac{140 \cdot 125}{300} = 58,3 \quad E_{23} = 140 - 32,7 - 58,3 = 49$$

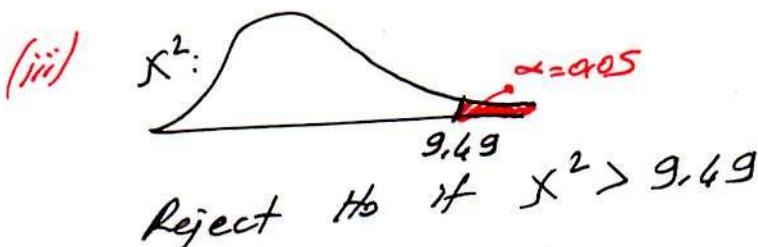
$$E_{31} = \frac{120 \cdot 70}{300} = 28 \quad E_{32} = \frac{120 \cdot 125}{300} = 50 \quad E_{33} = 120 - 28 - 50 = 42$$

(i)  $H_0$ : Family Size and Size of Washing Machine are independent

$H_A$ : There exists a relationship between them

$$\alpha = 0,05$$

(ii)  $\chi^2 = \sum \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$ ; d.o.f =  $(3-1) \cdot (3-1) = 4$



(iv)  $\chi^2 = \frac{(25-9,3)^2}{9,3} + \frac{(10-16,7)^2}{16,7} + \dots + \frac{(59-42)^2}{42} = 58,43$

(v) Reject  $H_0$ . Size of Wash. Mach. changes w.r. to family size.



## ANOVA

\* We've seen an application of ANOVA at Multiple Regression for Validity of the model test. We were testing  $H_0: \beta_1 = \beta_2 = \dots = \beta_k = 0$ .

\* Remember the two sample mean test:  $H_0: \mu_1 = \mu_2$ . The test statistics was  $t$  for unknown variance case.

\* Here, we will use ANOVA to test

(i)  $H_0: \mu_1 = \mu_2 = \dots = \mu_k$

$H_A$ : At least one mean is different.

The idea is to compare group means' variation with the variation within groups (like error) and reject  $H_0$  if variation between group means is significantly higher than within group (error) variation.

The test statistics is F-test here.

(ii) 
$$F = \frac{MS \text{ Groups}}{MS \text{ Error}}$$

 $\rightarrow$  d.o.f =  $k-1$ 

 $MSG = \frac{SSG}{k-1}$ 
  

 $\rightarrow$  d.o.f =  $n-k$ 

 $MSE = \frac{SSE}{n-k}$

The formula for SSG and SSE is described in the following simple example.

**Ex** Let 3 machines have following data for each 4 output: # of products per hour.



Machine 1	Machine 2	Machine 3	
6	11	1	
5	10	5	
8	10	7	
9	13	3	
<hr/>	<hr/>	<hr/>	$\bar{X} = 7,33$
$\bar{X}_1 = 7$	$\bar{X}_2 = 11$	$\bar{X}_3 = 4$	

The within sum of squares is Error sum of squares:

$$SSE = (6-7)^2 + (5-7)^2 + (8-7)^2 + (9-7)^2 + \overset{\text{Mach 1}}{+} \\ (11-11)^2 + (10-11)^2 + (10-11)^2 + (13-11)^2 + \overset{\text{Mach 2}}{+} \\ (1-4)^2 + (5-4)^2 + (7-4)^2 + (3-4)^2 = 36 \overset{\text{Mach 3}}{+}$$

$$SSE = \sum_{i=1}^k \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2$$

The Treatment of the machines is the deviation between their means and the grand mean. If all the observations were equal to their group's mean;

$$SSTREATMENT \text{ (or } SS_{GROUPS}) = 4 \cdot (7 - 7,33)^2 + \\ 4 \cdot (11 - 7,33)^2 + \\ 4 \cdot (4 - 7,33)^2 = 98$$

$$SSTB = \sum n_i \cdot (\bar{X}_i - \bar{X})^2$$

Total sum of squares is the deviation between each observation and the grand mean.

$$SSTOTAL = (6-7,33)^2 + (5-7,33)^2 + \dots + (3-7,33)^2 = 134$$



$$SST = \sum_{i=1}^K \sum_{j=1}^{n_i} (X_{ij} - \bar{X})^2$$

Observe that SST is the total of SSE and SST<sub>A</sub>. This is a general rule, not a coincidence.

$$SST = SST_A + SSE$$

These formulas were the "definitional formulas" to give the idea of ANOVA. We have shorter (and so, easier) formulas for sum of squares which gives the same results. The calculation formulas are as follows;

	Machine 1	Machine 2	Machine 3	
	6	11	1	
$n_1 = 4$	5	10	5	$n_2 = 4$
	8	10	7	$n_3 = 4$
	9	13	3	
<hr/>				
	$T_1 = \sum_{i=1}^4 X_{1i} = 28$	$T_2 = \sum_{i=1}^4 X_{2i} = 44$	$T_3 = \sum_{i=1}^4 X_{3i} = 16$	$T = T_1 + T_2 + T_3 = 88$
				$n = n_1 + n_2 + n_3 = 12$

$$\sum \sum X_{ij}^2 = 6^2 + 5^2 + \dots + 3^2 = 780$$

$$(i) \quad SST = \sum_{i=1}^K \sum_{j=1}^{n_i} X_{ij}^2 - \frac{T^2}{n} = 780 - \frac{88^2}{12} = 134$$

$$(ii) \quad SST_A = \sum_{i=1}^K \frac{T_i^2}{n_i} - \frac{T^2}{n} = \left( \frac{28^2}{4} + \frac{44^2}{4} + \frac{16^2}{4} \right) - \frac{88^2}{12} = 98$$

$$(iii) \quad SSE = SST - SST_A = 134 - 98 = 36$$

The term  $\frac{T^2}{n}$  is called correction term;  $CT = \frac{T^2}{n}$

The Hypothesis Test that "Machine output means are the same" versus "At least one mean is different" is as follows;

(i)  $H_0: \mu_1 = \mu_2 = \mu_3$

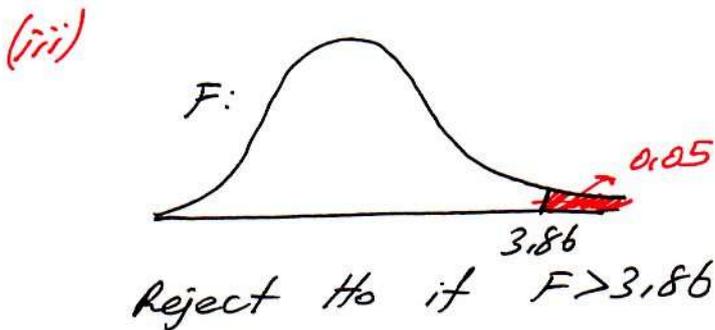
$H_A: \text{At least one } \mu_i \text{ is different, } i=1,2,3$

$\alpha = 0,05$

(ii)  $F = \frac{MSTR}{MSE}$

$\rightarrow \text{d.o.f} = k-1 = 3-1 = 2$

$\rightarrow \text{d.o.f} = n-k = 12-3 = 9$



(iv) ANOVA TABLE

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
Treatment	$SST_A = 98$	$k-1 = 3-1 = 2$	$MSTR = \frac{SST_A}{k-1} = \frac{98}{2} = 49$	$F = \frac{MSTR}{MSE} = \frac{49}{4} = 12,25$
Error	$SSE = 36$ $SSE = 134 - 98$	$n-k = 12-3 = 9$ ( $11-2 = 9$ )	$MSE = \frac{SSE}{n-k} = \frac{36}{9} = 4$	—
Total	$SST = 134$	$n-1 = 12-1 = 11$	—	—

(v) Reject  $H_0$ . At least one machine's output is different at  $\alpha = 0,05$

17.5 An instructor has a class of 23 students. At the beginning of the semester each student is randomly assigned to one of four teaching assistants—Smiley, Haydon, Alleline, or Bland. The students are encouraged to meet with their assigned teaching assistant to discuss difficult course material. At the end of the semester a common examination is administered. The scores obtained by students working with these teaching assistants are shown in the accompanying table.

Smiley	Haydon	Alleline	Bland
72	78	80	79
69	93	68	70
84	79	59	61
76	97	75	74
64	88	82	85
	81	68	63

- Calculate the within-groups, between-groups, and total sum of squares.
- Complete the analysis of variance table, and test the null hypothesis of equality of population mean scores for the teaching assistants.

17.5)

Smiley (I)	Haydon (II)	Alleline (III)	Bland (IV)
72	78	80	79
69	93	68	70
$n_1=5$ 84	$n_2=6$ 79	$n_3=6$ 59	$n_4=6$ 61
76	97	75	74
64	88	82	85
	81	68	63

$$T_1 = 365 \quad T_2 = 516 \quad T_3 = 432 \quad T_4 = 432$$

$$T = 365 + 516 + 432 + 432 = 1745$$

$$n = 5 + 6 + 6 + 6 = 23$$

$$\sum_{i=1}^4 \sum_{j=1}^{n_i} x_{ij}^2 = 72^2 + 69^2 + \dots + 63^2 = 136571$$

$$CT = \frac{T^2}{n} = \frac{1745^2}{23} = 132392,4$$

$$SST = 136571 - 132392,4 = 2178,6$$

$$SSTR = \frac{365^2}{5} + \frac{516^2}{6} + \frac{432^2}{6} + \frac{432^2}{6} - 132392,4 = 836,6$$

$$SSE = 2178,6 - 836,6 = 1342$$

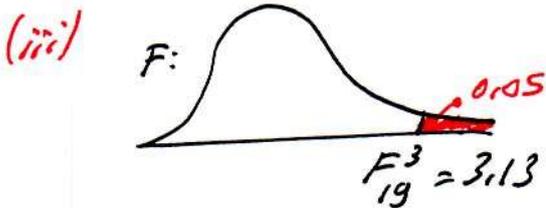
*ANCOVA CRABLE*

(i)  $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$   
 $H_A: \text{At least one } \mu_i \text{ is different}$

$$\alpha = 0,05$$

(ii)  $F = \frac{MSTR}{MSE}$

$\rightarrow d.o.f = 4 - 1 = 3$   
 $\rightarrow d.o.f = 23 - 4 = 19$



Reject  $H_0$  if  $F > 3,13$

Source	S.S.	d.o.f.	M.S.	F
Treatment	836,6	4-1=3	$\frac{836,6}{3} = 279$	$\frac{279}{71} = 3,93$
Error	$2178,6 - 836,6 = 1342$	22-3=19	$\frac{1342}{19} = 71$	
Total	2178,6	23-1=22		

(v) Reject  $H_0$ .

(74)

## KRUSKAL - WALLIS TEST

\* Kruskal-Wallis Test is a non-parametric test for one-way Anova. The usual one-way Anova testing procedure assumes the two populations have normal distribution with equal variance. Remember that nonparametric tests are distribution-free tests. Kruskal Wallis test is based on Ranks. The procedure is as follows;

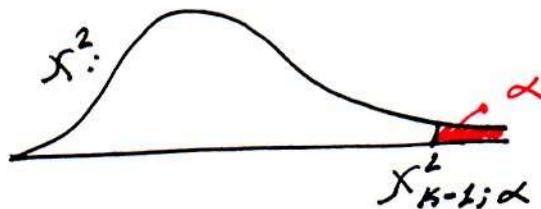
(i) Pool the data and Rank in ascending order. Write each datum's Rank.

(ii) Test statistics of Kruskal-Wallis is;

$$W = \frac{12}{n(n+1)} \cdot \sum_{i=1}^k \frac{R_i^2}{n_i} - 3 \cdot (n+1)$$

$W \underset{\text{app.}}{\sim} \chi^2_{(k-1)}$  where  $R_i^2$  is total of the Rank's square for group  $i$ .

(iii)



Reject  $H_0$  if  $W > \chi^2_{k-1, \alpha}$

(iv) Calculate  $W$  and reach a decision.

17.9) Using the data of Exercise 17.5, perform a Kruskal-Wallis test.

I	Rank	(II)	Rank	(III)	Rank	(IV)	Rank
Similey		Haydon		Allaine		Bland	
72	9	78	13	80	16	79	14.5
69	7	93	22	68	5.5	70	8
84	19	79	4.5	59	1	61	2
76	12	97	23	75	11	74	10
64	4	88	21	82	18	85	20
		81	17	68	5.5	63	3
	$R_1 = 365$		$R_2 = 516$		$R_3 = 305$		$R_4 = 432$

Pooled Data

59	61	63	64	68	68	69	70	72	74	75	76
1	2	3	4	5.5	5.5	7	8	9	10	11	12

Pooled Data

78	79	79	80	81	82	84	85	88	93	97
13	14.5	14.5	16	17	18	19	20	21	22	23

$n = 23$

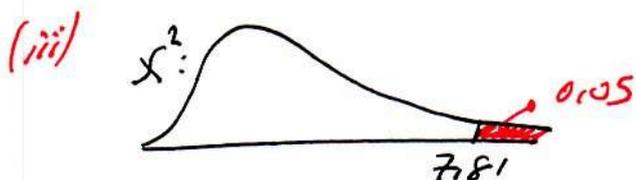
(i)  $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_A$ : At least one  $\mu_i$  is different

$\alpha = 0.05$

(ii)  $W = \frac{12}{n \cdot (n-1)} \sum_{i=1}^4 \frac{R_i^2}{n_i} - 3 \cdot (n+1)$

$W \sim \chi^2_{(3)}$



Reject  $H_0$  if  $\chi^2 > 7.81$

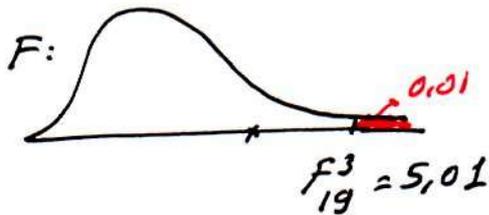
(iv)  $W = \frac{12}{23 \cdot 24} \cdot \left( \frac{365^2}{5} + \dots + \frac{432^2}{6} \right)$

$W = 2656.025$

(v) Reject  $H_0$

\* Note that at  $\alpha = 0,05$ , both F-test and Kruskal Wallis test have the same result. However, at  $\alpha = 0,01$ , the results of tests differ.

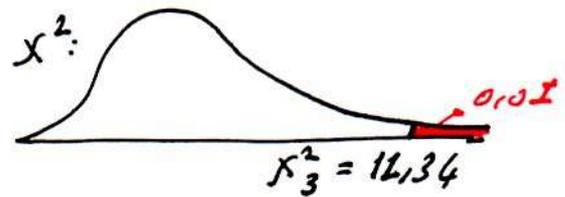
F-test



$$F = 3,93$$

Do NOT Reject  $H_0$ .

Kruskal-Wallis test



$$W = 2456,025$$

Reject  $H_0$ .

The F-test's p-value is between 0,01 and 0,05. Kruskal Wallis p-value is almost 0.

The reason is that, if we consider grades, they may be similar, at  $\alpha = 0,01$  resulting no significant difference. If Normality and Equality of variances assumptions are true, this result is reliable.

However, when we consider Peaks, Hayden's students have very bad ranks. If the assumptions are NOT correct, Kruskal Wallis test is more reliable and I should re-consider Hayden's performance seriously.

## Two-Way ANOVA: Block Design.

\* In Two way anova, we have two factors to be considered. The table of Block-Design is a contingency table where a variable is in rows and another variable is in columns.

	Group				TOTAL
Block	1	2	...	K	
1	$x_{11}$	$x_{12}$	...	$x_{1K}$	$T_{1.}$
2	$x_{21}$	$x_{22}$	...	$x_{2K}$	$T_{2.}$
...	...	...	...	...	...
H	$x_{H1}$	$x_{H2}$	...	$x_{HK}$	$T_{H.}$
TOTAL	$T_{.1}$	$T_{.2}$	...	$T_{.K}$	$T_{..}$

We have two hypothesis;

(I)  $H_0$ : Block Means are equal  
 $H_A$ : At least one Block Mean is different

(II)  $H_0$ : Group Means are equal  
 $H_A$ : At least one group mean is different.

\* The calculations are similar to that of one way ANOVA. The difference is, we add  $SS_{Blocks}$

$$SSB = \sum \frac{T_{i.}^2}{K} - CT$$

and  $SSE = SST - SSTA - SSB$

We find two F values to be compared with appropriate degrees-of-freedom F-table values.

17.39 Three television pilot shows for potential situation comedy series were shown to audiences in four regions of the country—the East, the South, the Midwest, and the West Coast. Based on audience reactions, a score (on a scale from 0 to 100) was obtained for each show. The sums of squares between groups (shows) and between blocks (regions) were found to be

$$SSG = 95.2 \quad \text{and} \quad SSB = 69.5$$

and the error sum of squares was

$$SSE = 79.3$$

Set out the analysis of variance table, and test the null hypothesis that the population mean scores for audience reactions are the same for all three shows.

$$K = 3 \text{ Shows; } H = 4 \text{ Regions}$$

$$n = 3 \cdot 4 = 12$$

ANOVA table

Source	SS	d.o.f.	MS	F
Treatment (Shows)	95.2	$K-1 = 3-1=2$	$\frac{95.2}{2} = 47.75$	$F_{TA} = \frac{47.75}{13.22} = 3.612$
Block (Regions)	69.5	$H-1 = 4-1=3$	$\frac{69.5}{3} = 23.17$	$F_B = \frac{23.17}{13.22} = 1.753$
Error	79.3	$11-2-3 = 6$	$\frac{79.3}{6} = 13.22$	
Total	244	$n-1 = 12-1=11$		

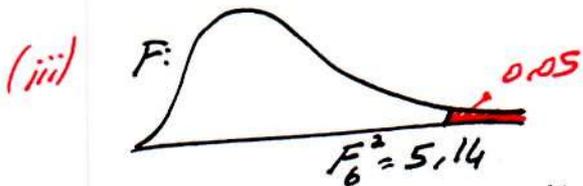
Shows (Treatment)

(i)  $H_0: \mu_1 = \mu_2 = \mu_3$

$H_A$ : At least one  $\mu_i$  is different

$$\alpha = 0.05$$

(ii)  $F_{TA} = \frac{MSTA}{MSE}$    
 $\rightarrow$  d.o.f = 2   
 $\rightarrow$  d.o.f = 6



Reject  $H_0$  if  $F > 5.14$

(iv)  $F_{TA} = 3.612$

(v) Do NOT Reject  $H_0$ . Audience reactions to shows are NOT significantly different at  $\alpha = 0.05$ .

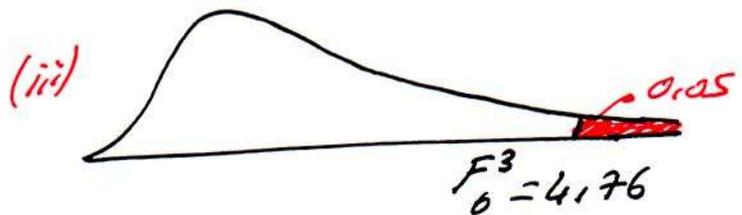
Regions (Block) (Not asked in the question)

(i)  $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_A$ : At least one  $\mu_i$  is different

$$\alpha = 0.05$$

(ii)  $F_B = \frac{MSB}{MSE}$    
 $\rightarrow$  d.o.f = 3   
 $\rightarrow$  d.o.f = 6



Reject  $H_0$  if  $F > 4.76$

(iv)  $F = 1.753$

(v) Do NOT Reject  $H_0$ . Audience reactions do NOT differ at different regions at  $\alpha = 0.05$

17.31 An agricultural experiment designed to assess differences in yields of corn for four different varieties, using three different fertilizers, produced the results (in bushels per acre) shown in the table.

Fertilizer	Variety			
	A	B	C	D
1	86	88	77	84
2	92	91	81	93
3	75	80	83	79

- Set out the two-way analysis of variance table.
- Test the null hypothesis that the population mean yields are identical for all four varieties of corn.
- Test the null hypothesis that population mean yields are the same for all three brands of fertilizer.

Fertilizer	Variety				Total
	A	B	C	D	
1	86	88	77	84	$T_1 = 335$
2	92	91	81	93	$T_2 = 357$
3	75	80	83	79	$T_3 = 317$
Total	$T_A = 253$	$T_B = 259$	$T_C = 241$	$T_D = 256$	$T_{..} = 1009$

$$n = 3 \cdot 4 = 12$$

$$C.T = \frac{1009^2}{12} = 84860,08$$

$$\sum \sum X_{ij}^2 = 86^2 + 92^2 + \dots + 79^2 = 85235$$

$$SST = 85235 - 84860,08 = 374,92$$

$$SST_A = \frac{253^2}{3} + \dots + \frac{256^2}{3} - 84860 = 62$$

$$SSB = \frac{335^2}{4} + \frac{357^2}{4} + \frac{317^2}{4} - 84860 = 201$$

$$SSE = 374,92 - 62 - 201 = 111,92$$

(i)  $H_{03}: \mu_A = \mu_B = \mu_C = \mu_D$

$H_{A3}$ : At least one mean is different

$$\alpha = 0,05$$

$H_{0II}: \mu_1 = \mu_2 = \mu_3$

$H_{AII}$ : At least one mean is different

ANOVA table

Source	S.S	d.o.f	MS	F
Treatment	62	$4-1=3$	$62:3 = 20,7$	$20,7:22 < 1$
Block	201	$3-1=2$	$201:2 = 100,5$	$100,5:22 = 4,57$
Error	132	$11-3-2=6$	$132:6 = 22$	
Total	395	$12-1=11$		

Treatment effect is NOT significant  
Block effect is NOT significant.