DECISION ANALYSIS for BIM LECTURE NOTES | QUALITY CONTROL DECISION ANALYSIS

QUALITY CONTROL

Quality Control is a very important aspect of production. Every production process has a Quality Control departpest. For example, consider 1 lt Cola production. It is familier that, in cola bottles, valume is represented by 1 lt ± 5cl. The specification limits 995cl and 1005cl are controlled by Statistical Quality Control.

Consider a production process. There are two kinds of variability, which are;

(i) Natural (or Random) Variation

(ii) Assignable Variation.

Our interest is if second kind of variation occurs, due to machine fault, raw material mischoose, worker fault etc. If there's NO such a cause, we say that Process is "in control." However, if such a cause, lets say shift the mean (for example 1003cl production of Cola), we say that the process is "out of control."

The process mean is controled by "$\bar{x}$-chart" and the process variation is controled by ranges: "R-chart"

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$\bar{X}$ and R charts (Control Charts)

Typically, a random sample of size n is taken from the process periodically (every hours, every week etc...) for k periods and for each period, $\bar{X}_i$ and R_i values are calculated. The process is considered to be in control if the data values $\bar{X}_i$ and R_i are within control limits.

* For an $\bar{X}$ chart, the control limits are;

$$UCL = \bar{\bar{X}} + 3 \cdot \frac{\bar{R}}{d_2\sqrt{n}} \quad \text{and} \quad LCL = \bar{\bar{X}} - 3 \cdot \frac{\bar{R}}{d_2\sqrt{n}}$$

Upper control limit — Lower control limit

where, the central line is $CL = \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k}$

and $\bar{R}$ is the mean range of data; $\bar{R} = \frac{\sum R_i}{k}$

* For an R chart, the control limits are;

$$UCL = \bar{R}\left(1 + 3 \cdot \frac{d_3}{d_2}\right) \quad \text{and} \quad LCL = \bar{R}\left(1 - 3 \cdot \frac{d_3}{d_2}\right)$$

where, the central line is $\bar{R}$, d_2 and d_3 are table values that will be given.

* Other commonly used charts are, p-chart (proportion of defective items) and c-chart (number of nonconforming items). Note that, since proportion and number can NOT be negative, if their lower limits are found negative, we take them 0. We also take the LCL of R-chart 0 if it is negative.

■ TABLE 6E.3
Bearing Diameter Data

Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
1	34.5	3	13	35.4	8
2	34.2	4	14	34.0	6
3	31.6	4	15	37.1	5
4	31.5	4	16	34.9	7
5	35.0	5	17	33.5	4
6	34.1	6	18	31.7	3
7	32.6	4	19	34.0	8
8	33.8	3	20	35.1	4
9	34.8	7	21	33.7	2
10	33.6	8	22	32.8	1
11	31.9	3	23	33.5	3
12	38.6	9	24	34.2	2

6.3. The data shown in Table 6E.3 are $\bar{x}$ and R values for 24 samples of size $n = 5$ taken from a process producing bearings. The measurements are made on the inside diameter of the bearing, with only the last three decimals recorded (i.e., 34.5 should be 0.50345). $d_2 = 2,326$; $d_3 = 0,864$

(a) Set up $\bar{x}$ and R charts on this process. Does the process seem to be in statistical control? If necessary, revise the trial control limits.

b) Are the following values in control or out of control (OR outlier)?

$\bar{X}_i = 33,8$ $\bar{X}_i = 29,4$ $\bar{X}_i = 38,5$

$R_i = 3$ $R_i = 8$ $R_i = 11$

6.3) a) For $\bar{X}$ chart, $n = 5$; $k = 24$; $\bar{R} = \frac{\Sigma R_i}{k} = \frac{3+6+4+\dots+3+2}{24} = 4,71$

$$CL = \bar{\bar{X}} = \frac{\Sigma \bar{X}_i}{k} = \frac{34,5+36,2+34,6+\dots+33,5+36,2}{24} = 34,00$$

$$UCL = 34,00 + 3 \cdot \frac{4,71}{2,326 \cdot \sqrt{5}} = 36,72$$

$$LCL = 34,00 - 3 \cdot \frac{4,71}{2,326 \cdot \sqrt{5}} = 31,29$$

Note that, sample No. 12 and 15 are out of control because they are greater than UCL.

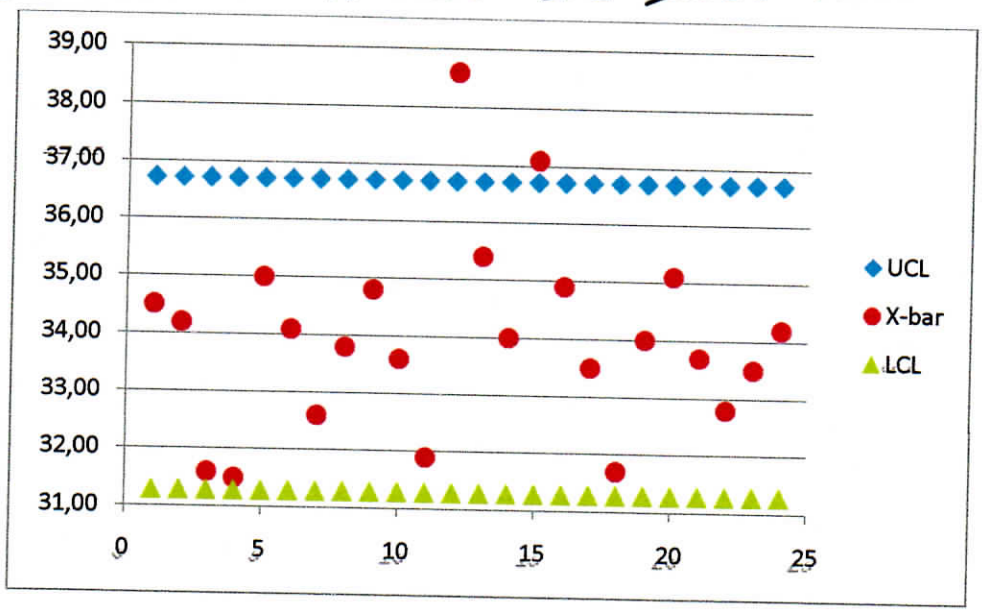
For R chart,

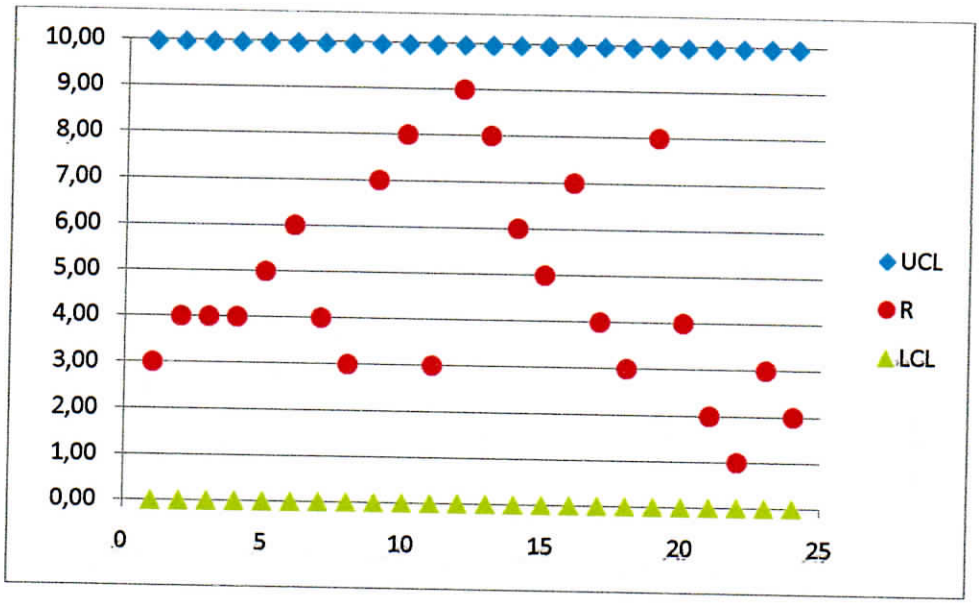
$$CL = \bar{R} = 4,71$$

$$UCL = 4,71 \cdot \left(1 + 3 \cdot \frac{0,864}{2,326}\right) = 9,96$$

$$LCL = 4,71 \cdot \left(1 - 3 \cdot \frac{0,864}{2,326}\right) = -0,54 < 0 \Rightarrow \text{Take } LCL = 0$$

The $\bar{X}$-chart and R-chart are given below.





b) $\bar{x}_i = 33{,}8$ is within the control limits. However, $\bar{x}_i = 29{,}4$ (below LCL) and $\bar{x}_i = 38{,}5$ (above UCL) are out of control (outliers)

$R_i = 3$ and $R_i = 8$ are within control limits. However, $R_i = 11$ is an outlier. Then, the process is out of control.

Acceptance Sampling

* From a large lot, we take a random sample of size n and observe the number of defective items: D. If the number of defective items is at most c, we accept the whole lot. Otherwise, we reject the lot. This is called "single sampling scheme"

For example, let $n=100$ and $c=2$. This means;

(i) Select a sample of size $n=100$

(ii) Accept the whole lot if $D \leq 2$ (0, 1 or 2 defectives)

Let, the (unknown) proportion of defectives is $p=0,015$. Then, D has a Binomial distribution. However, we'll approximate Binomial Probabilities by Poisson Distribution. Then;

$$\mu = n \cdot p = 100 \cdot 0,0015 = 1,5$$

$$D \sim \text{Poisson}(\mu = 1,5)$$

$$f(d) = \frac{e^{-\mu} \cdot \mu^d}{d!} = \frac{e^{-1,5} \cdot 1,5^d}{d!}$$

Probability of accepting the lot is called "Consumer's Risk" and rejecting the lot is called "Producer's Risk". Then;

$$\text{Consumer's Risk} = P(\text{Accept}) = P(D \leq 2) = f(0) + f(1) + f(2)$$

$$= e^{-1,5}\left(\frac{1,5^0}{0!} + \frac{1,5^1}{1!} + \frac{1,5^2}{2!}\right) = 0,809$$

$$\text{Producer's Risk} = P(\text{Reject}) = 1 - 0,809 = 0,191$$

* The double-stage sampling is as follows.

Given n_1, n_2, c_1 and c_2;

step 1 { (i) Select a random sample of size n_1
(ii) If $D_1 \leq c_1$, accept the lot.
If $D_1 > c_2$, reject the lot.
If $c_1 < D_1 \leq c_2$, GOTO step 2

step 2 { (iii) Select another random sample of size n_2
(iv) If $D_1 + D_2 \leq c_2$, accept the lot.
If $D_1 + D_2 > c_2$, Reject the lot.

Note that, at second stage we consider "TOTAL number of defectives"

15.13. Draw the primary and supplementary OC curves for a double-sampling plan with $n_1 = 50$, $c_1 = 2$, $n_2 = 100$, $c_2 = 6$. If the incoming lots have fraction non-conforming $p = 0.05$,

a) WPT lot is accepted in the first sample?

b) If there are 5 defectives in the first sample, WPT lot is accepted in the second sample?

15.13) $n_1 = 50$ $c_1 = 2$; $p = 0,05$
$n_2 = 100$ $c_2 = 6$

a) First sample; $n_1 = 50$; $\mu_1 = 50 . 0,05 = 2,5$

Accept if $D_1 \leq 2$
Reject if $D_1 > 6$
GOTO Second Sample if $2 < D \leq 6$

Then,

$$P(\text{Accept}) = P(D_1 \leq 2) = f_1(0) + f_1(1) + f_1(2)$$
$$= e^{-2,5}\left(\frac{2,5^0}{0!} + \frac{2,5^1}{1!} + \frac{2,5^2}{2!}\right) = 0,544$$

b) Second Sample; $n_2 = 100$
$\mu_2 = 100 . 0,05 = 5$

Accept the lot if $D_1 + D_2 \leq 6$
Reject the lot if $D_1 + D_2 > 6$

Then, $P(\text{Accept}) = P(D_1 + D_2 \leq 6 \mid D_1 = 5) = P(5 + D_2 \leq 6) = P(D_2 \leq 1)$

$$= f_2(0) + f_2(1) = e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!}\right) = 0,0404$$

DECISION ANALYSIS

Consider a case we have several alternatives, but the utility (ie. gain) of the alternatives are uncertain. Statistical Decision Analysis is used to make "best choice" among several alternatives. Our decisions will be based on "Expected Values" of the choices.

Optimum Stock Size

15.2-2. Jean Clark is the manager of the Midtown Saveway Grocery Store. She now needs to replenish her supply of strawberries. Her regular supplier can provide as many cases as she wants. However, because these strawberries already are very ripe, she will need to sell them tomorrow and then discard any that remain unsold. Jean estimates that she will be able to sell 10, 11, 12, or 13 cases tomorrow. She can purchase the strawberries for \$3 per case and sell them for \$8 per case. Jean now needs to decide how many cases to purchase.

Jean has checked the store's records on daily sales of strawberries. On this basis, she estimates that the prior probabilities are 0.2, 0.4, 0.3, and 0.1 for being able to sell 10, 11, 12, and 13 cases of strawberries tomorrow.

15.2-2) We have the demand distribution;

Demand	P(Demand)
10	0,2
11	0,4
12	0,3
13	0,1

Purchase = \$3 ; Sell = \$8.

The profit of purchasing a strawbery is;

Marginal Profit = MP = 8 - 3 = \$5

If a strawbery is unsold, its loss is;

Marginal Loss = ML = \$3

* Consider the following "Profit Table"

Demand	Stock 10	11	12	13	P(Demand)
10	50	47	44	41	0,2
11	50	55	52	49	0,4
12	50	55	60	57	0,3
13	50	55	60	65	0,1

Note that; if Stock ≤ Demand, Profit = 5. Stock.

if Stok > Demand, Profit = 5. Demand
−3. (stock − Demand)

The expected profit of stocking s strawberies are;

$E(s=10) = 50.0,2 + 50.0,4 + 50.0,3 + 50.0,1 = 50$

$E(s=11) = 47.0,2 + 55.0,4 + 55.0,3 + 55.0,1 = 53,4$

$E(s=12) = 44.0,2 + 52.0,4 + 60.0,3 + 60.0,1 = 53,6$ ↳ Maximum

↳ Optimum stock size

$E(s=13) = 41.0,2 + 49.0,4 + 57.0,3 + 65.0,1 = 51,4$

Optimum Stock site is 12 since its Expected Profit 53,6 is the maximum one.

* Note that, the diagonal values of the Profit Table are the maximum values of each row. The idea is, "if we knew the demand, we would stock that demand and gain maximum profit." An "opportunity loss" is the difference between Profit and "maximum possible Profit."

Consider the following "Oportunity loss Table"

	Stocks				
Demand	10	11	12	13	P(Demand)
10	0	3	6	9	0,2
11	5	0	3	6	0,4
12	10	5	0	3	0,3
13	15	10	5	0	0,1

The expected opportunity loss of stocking s strawberies are;

$E'(s=10) = 0.0{,}2 + 5.0{,}6 + 10.0{,}3 + 15.0{,}1 = 6{,}5$

$E'(s=11) = 3.0{,}2 + 0.0{,}6 + 5.0{,}3 + 10.0{,}1 = 3{,}1$

$E'(s=12) = 6.0{,}2 + 3.0{,}6 + 0.0{,}3 + 5.0{,}1 = \boxed{2{,}9}$ → Minimum

↳ Optimum stock size

$E'(s=13) = 9.0{,}2 + 6.0{,}6 + 3.0{,}3 + 0.0{,}1 = 5{,}1$

This is an alternative solution to the "optimum stock size" problem of this kind. It is NOT coincidence that $s=12$ gives both Maximum Profit and Minimum Opportunity Loss Expected Values.

* Let, I know the demand everyday "With Certainty" and I want to sell you this information. What is the maximum price that you will pay me in order to increase your profit "in the long run"? (Namely Expected Profit)

The answer is "Expected Value of Perfect Information" (EVPI) If the demand is known with certainty, the profit will be the diagonal values of Profit Table.

Then; $E(\text{Maximum Profit}) = 50.0{,}2 + 55.0{,}6 + 60.0{,}3 + 65.0{,}1 = 56{,}5$

$E(\text{Optimum Profit}) = E(s=12) = 53{,}6$

$EVPI = E(\text{Maximum Profit}) - E(\text{Optimum Profit})$

$= 56{,}5 - 53{,}6 = 2{,}9$

Up to 2,9 $ you pay this "Perfect Information", your Profit will be higher than 53,9 in the long run.

* Fortunately, there is an easier way to find optimum stock size, which does not requires to make Profit or Opportunity loss table. The steps are as follows;

(i) Write the pmf of Demand in Descending Order and calculate Cummulative Sum of probabilities.

(ii) Calculate $\boxed{p^* = \frac{mL}{mP + mL}}$

(iii) The first Cummulative Sum you exceed (or equal) p^* corresponds to optimum stock size.

Applying these steps to our example; we have,

(i)

Demand	P(Demand)	Cum. Sum
13	0,1	0,1
12	0,3	0,4
11	0,4	0,8
10	0,2	1,0

(ii) $p^* = \frac{3}{3+5} = 0,375$

(iii) 0,4 "First time exceeds" 0,375. Then, optimum stock size is $s = 12$.

* A little bit complicated version of the last application occurs when we have "Salvage Value". Salvage Value is the Profit obtained by selling unsold items.

Let, for example, unsold strawberries are used to make a cake and sold (or given back) for 2,6 $. Then;

$$\left.\begin{aligned} mP &= 8 - 3 = 5 \\ mL &= 3 - 2,6 = 0,4 \end{aligned}\right\} \Rightarrow p^* = \frac{0,4}{5 + 0,4} = 0,074$$

And optimum stock size becomes $s = 13$.

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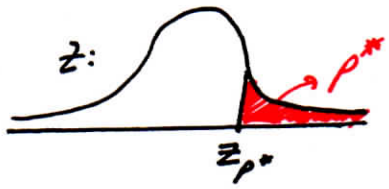
* If the demand has a Normal distribution, the application needs to be modified as follows;

(i) Find $\rho^* = \frac{ML}{MP+ML}$

(ii)



(iii) Since $z = \frac{x-\mu}{\sigma}$; Optimum Stock size is

$$S = z_{\rho^*} \cdot \sigma + \mu$$

Example: Daily demand for loaves of bread at a grocery store has a normal distribution with mean 185 and variance 3775. The store pays 55 cents a loaf and sells it for \$1,20 each. Any loaves that are not sold by the end of the day are disposed of at 25 cents each. Find the optimum stock size that maximizes daily profit.

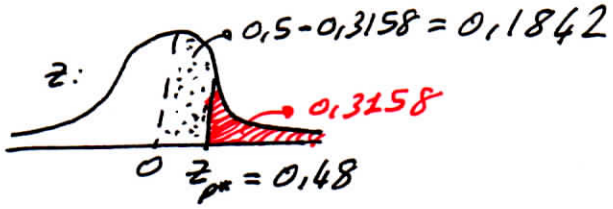
Answer: $D \sim Normal(\mu = 185 ; \sigma^2 = 3775)$

$\sigma = \sqrt{3775} = 61,44$

$MP = 1,20 - 0,55 = 0,65$

$ML = 0,55 - 0,25 = 0,30$

$\rho^* = \frac{0,30}{0,65+0,30} = 0,3158$



$S = 0,48 \cdot 61,44 + 185 = 214,29 \approx 214$ loaves

(Be careful, z_{ρ^*} may be Negative!)

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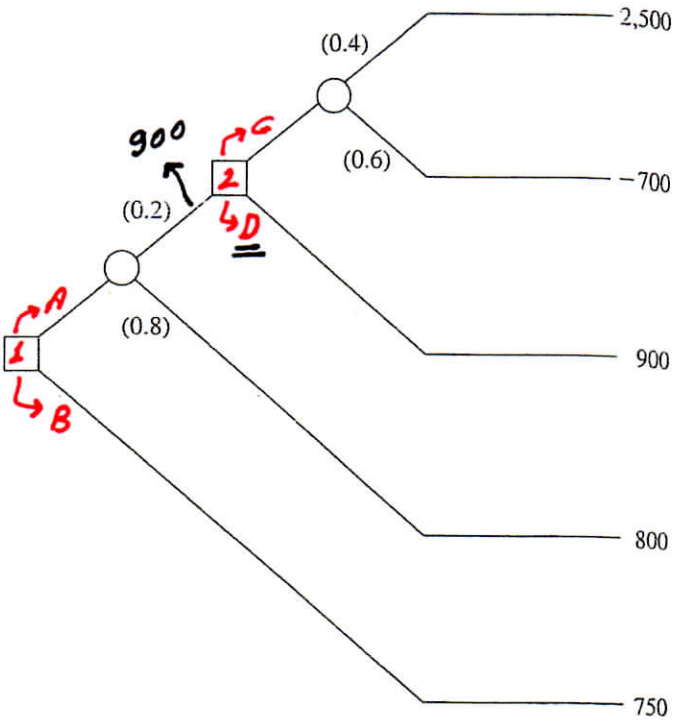
Decision Tree

Alternatives = □ ; Random events = ○

Choose the alternative that has higher expected Gain.

15.4-2. You are given the decision tree to the ~~data~~ where the numbers in parentheses are probabilities and the numbers on the far right are payoffs at these terminal points. Analyze this decision tree to obtain the optimal policy.



15.4-2) Start from the last decision!

$E(C) = 0,4 \cdot 2500 + 0,6 \cdot (-700) = 580$

$E(D) = 900$ (constant)

Choose D

Then;

$E(A) = 0,2 \cdot 900 + 0,8 \cdot 800 = 820$

$E(B) = 750$

Choose A

Solution; (Optimum Policy)

At the beginning (node 1) choose A. If the event with prob. 0,2 occurs, (node 2) choose D.

We are not always given the decision Tree explicitly. We need to make decision tree from the table given.

Alternatives are called "Actions" and random events are called "state of nature". Given a problem, keep in mind that state of natures have probabilities. This remarker make it easy to identify actions and state of nature, and to form a decision tree.

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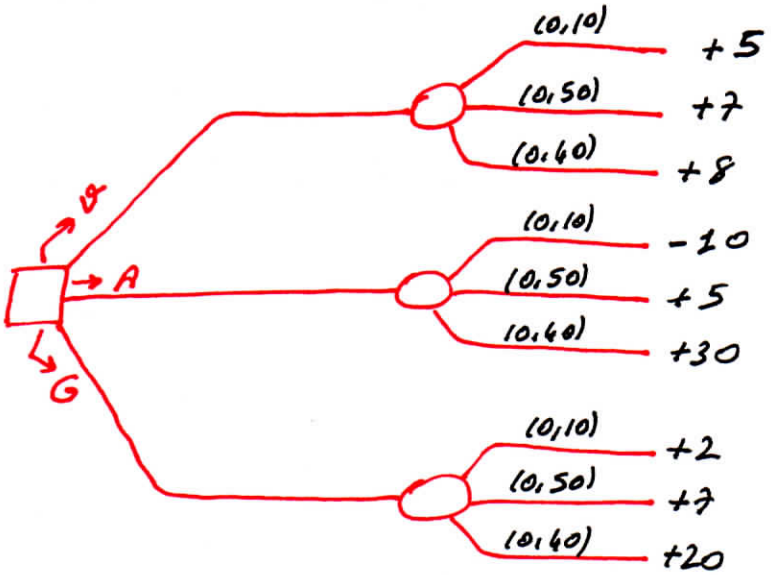
3. You have the chance to invest in three mutual funds: utility, aggressive growth, and global. The value of your investment will change depending on the market conditions. There is a 10% chance the market will go down, 50% chance it will remain moderate, and 40% chance it will perform well. The following table provides the percentage change in the investment value under the three conditions:

Alternative	Percent return on investment		
	Down market (%)	Moderate market (%)	Up market (%)
Utility	+5	+7	+8
Aggressive growth	−10	+5	+30
Global	+2	+7	+20

(a) Represent the problem as a decision tree.
(b) Which mutual fund should you select?

3.) Actions : U: Utility , A: Aggressive Growth, G: Global

States : P(Down) = 0,10, P(Moderate) = 0,50, P(Well) = 0,40



Select "Aggressive Growth" Mutual Fund since its expected gain is highest.

$E(U) = 0,10 \cdot 5 + 0,50 \cdot 7 + 0,40 \cdot 8 = 2,1$

$E(A) = 0,10 \cdot (-10) + 0,50 \cdot 5 + 0,40 \cdot 30 = 13,5$ → Maximum

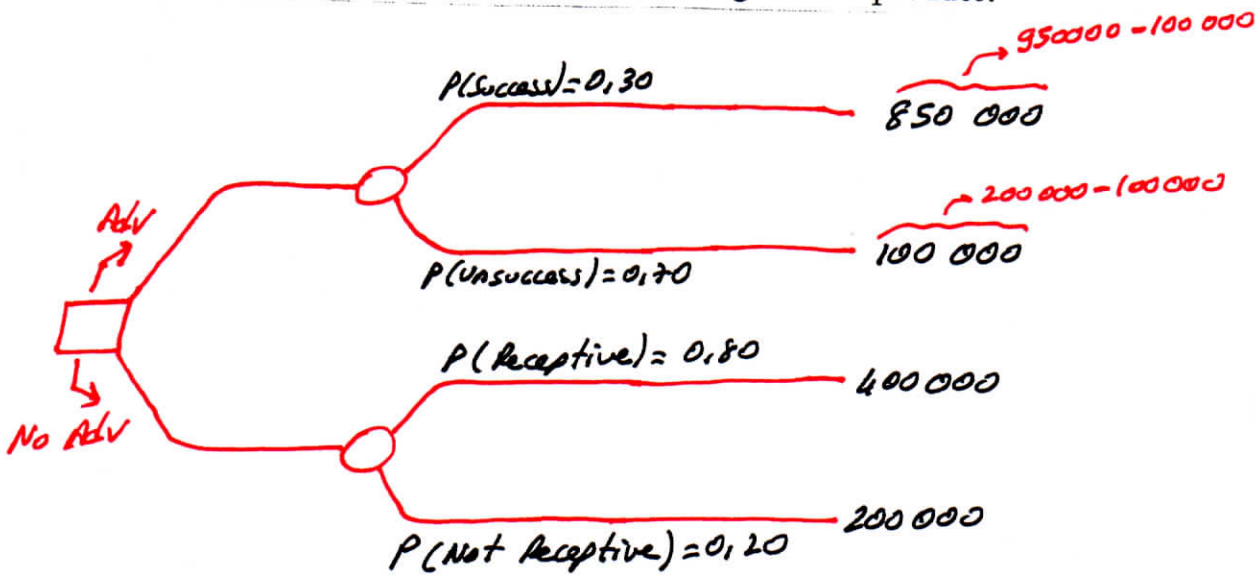
↳ choose Aggressive Growth

$E(G) = 0,10 \cdot 2 + 0,50 \cdot 7 + 0,40 \cdot 20 = 11,7$

5. AFC is about to launch its new Wings 'N Things fast food nationally. The research department is convinced that Wings 'N Things will be a great success and wants to introduce it immediately without advertisement in all AFC outlets. The marketing department sees "things" differently and wants to unleash an intensive advertisement campaign. The advertisement campaign will cost \$100,000 and if successful will produce \$950,000 revenue. If the campaign is unsuccessful (there is a 30% chance it won't be), the revenue is estimated at only \$200,000. If no advertisement is used, the revenue is estimated at \$400,000 with probability .8 if the customers are receptive and \$200,000 with probability .2 if they are not.

(a) Draw the associated decision tree.

(b) What course of action should AFC follow in launching the new product?



$E(Adv) = 0{,}30 \cdot 850\,000 + 0{,}70 \cdot 100\,000 = 325\,000$

$E(\text{No Adv}) = 0{,}80 \cdot 400\,000 + 0{,}20 \cdot 200\,000 = 360\,000$

Decide NOT to Make Advertisement since its Expected Revenue is higher.