QUALITY CONTROL LECTURE NOTES | CHAPTERS 5&6

* Control Charts monitor a process by taking random samples periodically. We control the process mean by $\bar{X}$ chart and process variability by R chart.

* We take a random Sample of size n for m periods. For an $\bar{X}$ chart, center line is set to mean of m sample means;

$$\text{Center line} = \bar{\bar{X}} = \frac{\sum_{i=1}^{m} \bar{X}_i}{m}$$

And the control limits are;

$$UCL = \bar{\bar{X}} + 3.\sigma_{\bar{x}}$$
$$LCL = \bar{\bar{X}} - 3.\sigma_{\bar{x}}$$

* Note that, (UCL; LCL) likes confidence interval for μ. The confidence level is $1-\alpha = P(-3 < z < 3) = 0,9973$. Namely, if the process is in control, 99,73% of the sample points is expected to lie within Control limits.

Each sample point is a Hypothesis test

$$H_0: \mu = \bar{\bar{X}}$$
$$H_A: \mu \neq \bar{\bar{X}}$$
$$\alpha = 0,0027$$

where we reject H_0 if a sample point falls outside the limits.

* Sometimes, the process may be out of control although all the sample means are within control limits. We call the interval $(\bar{\bar{x}} - 2\sigma ; \bar{\bar{x}} + 2\sigma)$ warning limits. The following sensitizing Rules are used to suspect an Out of Control Signal for $\bar{x}$ - Charts (OR Shewhart Charts)

(i) One or more points outside of the control limits

(ii) Two of the three consecutive points outside the two sigma warning limits but still inside the control limits

(iii) Four of five consequitive points beyond the one sigma limits.

(iv) A run of eight consequitive points on one side of the center line.

(v) Six points in a row steadily increasing or decreasing

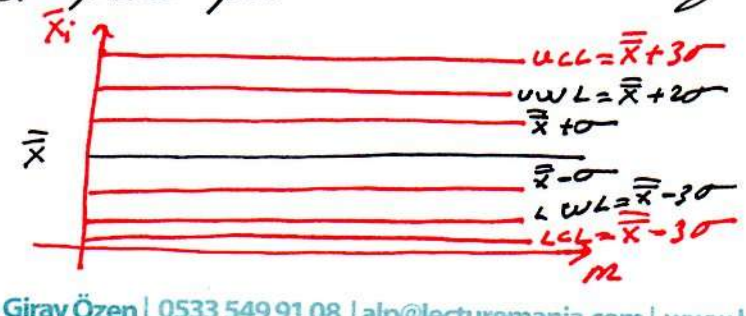
(vi) Fifteen points in a row within the one sigma limits.

(vii) Fourteen points in a row altering up and down.

(viii) Eight points in a row on both sides out the center line with none in one sigma limits.

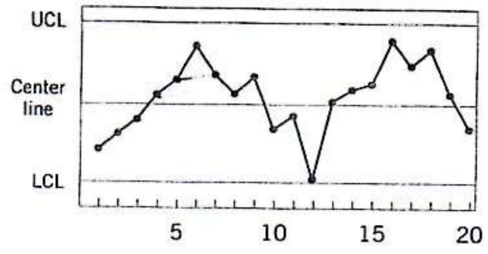
(ix) An unusual or nonrandom pattern in the data

(x) One or more points near a warning or control limit.

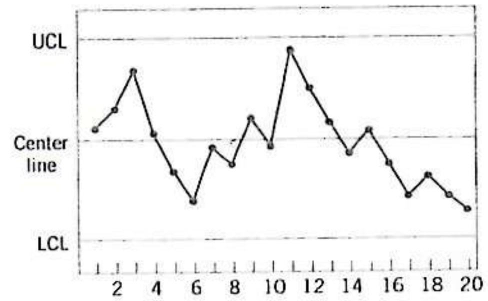


* Rules (i) - (iv) are called "Western Electric Rules"

5.9. Consider the control chart shown here. Does the pattern appear random?



5.10. Consider the control chart shown here. Does the pattern appear random?



5.9) There's a cyclic pattern, the control chart has a nonrandom pattern.

5.10) The pattern appear random.

5.18. Apply the Western Electric rules to the control chart presented in Exercise 5.9. Would these rules result in any out-of-control signals?

5.19. Consider the control chart shown in Exercise 5.10. Would the use of warning limits reveal any potential out-of-control conditions?

5.20. Apply the Western Electric rules to the control chart in Exercise 5.10. Are any of the criteria for declaring the process out of control satisfied?

5.18) Points 16, 17, 18 are 2 of 3 beyond 2σ of central line (Rule (ii)) AND Points 5, 6, 7, 8 and 9 are of 5 at 1σ or or beyond central line (Rule (iii))

5.19) Yes, points 16, 17, 18 are close to the upper warning limit (Rule (x))

5.20) Points 17, 18, 19 and 20 are outside the lower 1σ area (Rule (iii))

5.28. A process is normally distributed and in control, with known mean and variance, and the usual three-sigma limits are used on the $\bar{x}$ control chart, so that the probability of a single point plotting outside the control limits when the process is in control is 0.0027. Suppose that this chart is being used in phase I and the averages from a set of m samples or subgroups from this process is plotted on this chart. What is the probability that at least one of the averages will plot outside the control limits when $m = 5$? Repeat these calculations for the cases where $m = 10$, $m = 20$, $m = 30$, and $m = 50$. Discuss the results that you have obtained.

5.28) For $m=5$;

T: # of units outside the control limits.

$T \sim$ Binomial $(n = 5 ; p = 0{,}0027)$

$P(T \geq 1) = 1 - P(T=0) = 1 - 0{,}9973^5$
$= 0{,}0134$

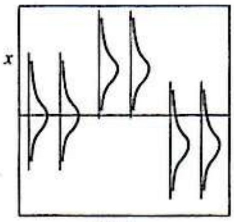
For $m=10$;

$P(T \geq 1) = 1 - 0{,}9973^{10} = 0{,}0267$

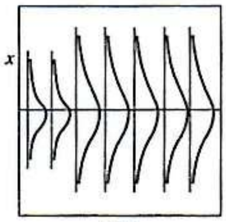
For $m=20$;

$P(T \geq 1) = 1 - 0{,}9973^{20} = 0{,}0526$

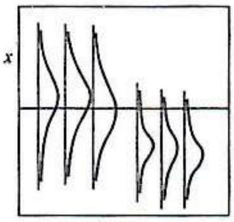
5.23. Consider the time-varying process behavior shown below. Match each of these several patterns of process performance to the corresponding $\bar{x}$ and R charts shown in figures (a) to (e) below.



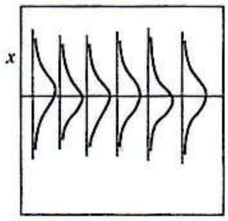
(a)



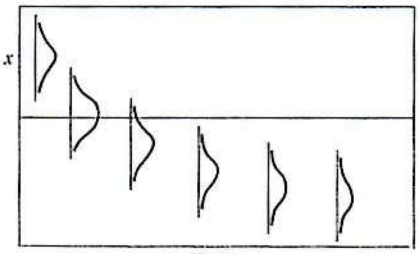
(b)



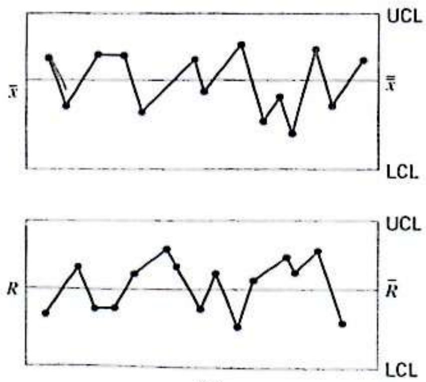
(c)



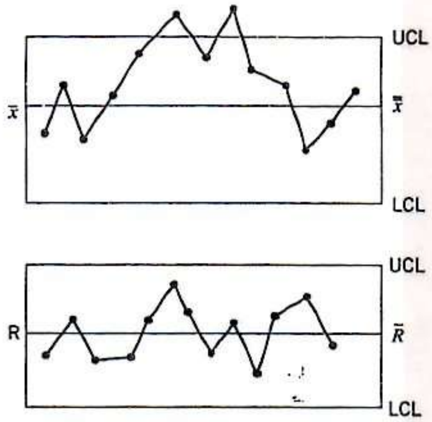
(d)



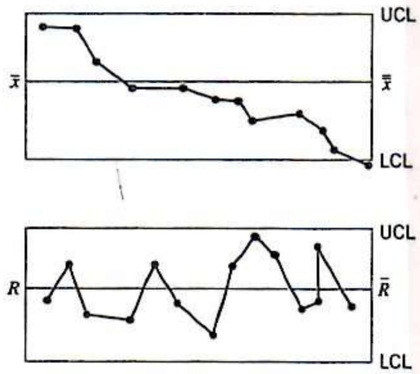
(e)



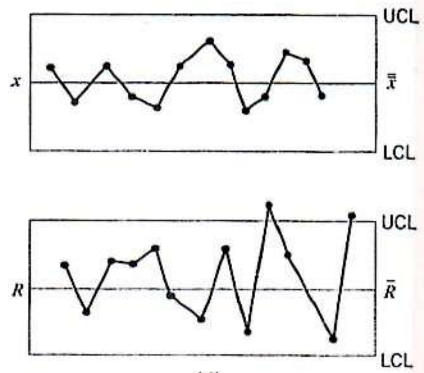
(1)



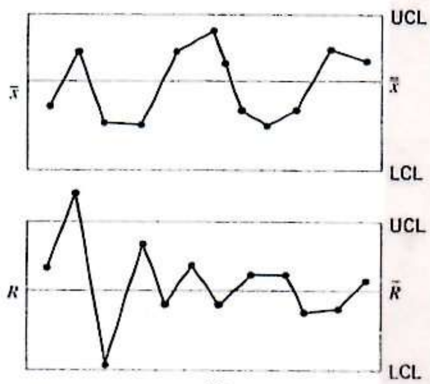
(2)



(3)



(4)



(5)

5.23) (a) → (2) ; (b) → (4) ; (c) → (5) ; (d) → (1) ; (e) → (3)

* A single measurable quality characteristic, such as a dimension, weight, or volume, is called a variable. So, the control charts for $\bar{X}$, R, s and s^2 are called variables charts.

$\bar{X}$ control chart controls the process average or mean.

R, s and s^2 control charts are used to control process variability.

Control charts for $\bar{X}$ and R

Let, for m periods, we have n observations for each.

We have; $\bar{x}_i = \frac{\sum x_i}{n}$; $R_i = \max(x_i) - \min(x_i)$

* Remember, $(1-\alpha).100\%$ C.I. for μ was,

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

We replace $\bar{x}$ with grand mean; $\bar{\bar{x}}$

$Z_{\alpha/2}$ by 3 because we usually use $\alpha = 0{,}0027$

σ is estimated by $\hat{\sigma} = \frac{\bar{R}}{d_2}$

where $\bar{\bar{x}} = \frac{\sum x_i}{m}$ and $\bar{R} = \frac{\sum R_i}{m}$.

Then, $(1-\alpha).100\%$ C.I. for μ is;

$$\bar{\bar{x}} \mp 3 . \frac{\bar{R}}{d_2\sqrt{n}}$$

Since we have $\frac{3}{d_2\sqrt{n}} = A_2$,

$(1-\alpha).100\%$ C.I for μ becomes

$$\bar{\bar{X}} \pm A_2 \cdot \bar{R} \qquad UCL = \bar{\bar{X}} + A_2\bar{R}$$
$$(LCL\ ;\ UCL) \qquad LCL = \bar{\bar{X}} - A_2\bar{R}$$

* Assuming that the quality characteristics is normally distributed; the standard deviation of $W = R/\sigma$; say d_3, is a function of n. Then,

$$\sigma_R = d_3 \cdot \sigma$$
$$\hat{\sigma}_R = d_3 \cdot \frac{\bar{R}}{d_2}$$

3σ control limits for the R chart is;

$$\bar{R} \pm 3\hat{\sigma}_R : \bar{R} \pm 3 d_3 \cdot \frac{\bar{R}}{d_2}$$

Then; $UCL = \bar{R} + 3d_3\dfrac{\bar{R}}{d_2} = \bar{R}\underbrace{\left(1 + \dfrac{3d_3}{d_2}\right)}_{=D_4} = D_4\bar{R}$

$$LCL = \bar{R} - 3d_3\frac{\bar{R}}{d_2} = \bar{R}\underbrace{\left(1 - \frac{3d_3}{d_2}\right)}_{=D_3} = D_3\bar{R}$$

* Control limits for $\bar{X}$-chart

$UCL = \bar{\bar{X}} + A_2\bar{R}$

Central Line $= \bar{\bar{X}}$

$LCL = \bar{\bar{X}} - A_2\bar{R}$

Estimated Process mean:
$\hat{\mu} = \bar{\bar{X}}$

* for R chart

$UCL = D_4\bar{R}$

Central line $= \bar{R}$

$LCL = D_3\bar{R}$

Estimated Process Std-Dev
$\hat{\sigma} = \dfrac{\bar{R}}{d_2}$

6.2. The net weight (in oz) of a dry bleach product is to be monitored by $\bar{x}$ and R control charts using a sample size of $n = 5$. Data for 20 preliminary samples are shown in Table 6E.2.

(a) Set up $\bar{x}$ and R control charts using these data. Does the process exhibit statistical control?

■ TABLE 6E.2
Data for Exercise 6.2.

Sample Number	x_1	x_2	x_3	x_4	x_5
1	15.8	16.3	16.2	16.1	16.6
2	16.3	15.9	15.9	16.2	16.4
3	16.1	16.2	16.5	16.4	16.3
4	16.3	16.2	15.9	16.4	16.2
5	16.1	16.1	16.4	16.5	16.0
6	16.1	15.8	16.7	16.6	16.4
7	16.1	16.3	16.5	16.1	16.5
8	16.2	16.1	16.2	16.1	16.3
9	16.3	16.2	16.4	16.3	16.5
10	16.6	16.3	16.4	16.1	16.5
11	16.2	16.4	15.9	16.3	16.4
12	15.9	16.6	16.7	16.2	16.5
13	16.4	16.1	16.6	16.4	16.1
14	16.5	16.3	16.2	16.3	16.4
15	16.4	16.1	16.3	16.2	16.2
16	16.0	16.2	16.3	16.3	16.2
17	16.4	16.2	16.4	16.3	16.2
18	16.0	16.2	16.4	16.5	16.1
19	16.4	16.0	16.3	16.4	16.4
20	16.4	16.4	16.5	16.0	15.8

(b) Estimate the process mean and standard deviation.

(c) Does fill weight seem to follow a normal distribution?

(d) If the specifications are at 16.2 ± 0.5, what conclusions would you draw about process capability?

(e) What fraction of containers produced by this process is likely to be below the lower specification limit of 15.7 oz?

6.2)

a) The necessary calculations are shown in the next page.

For $\bar{X}$ chart; $\bar{\bar{X}} = 16,27$

A_2 value for $n=5$ is; $0,577$

$UCL = \bar{\bar{X}} + A_2 \cdot \bar{R}$

$= 16,27 + 0,577 \cdot 0,47 = 16,54$

$LCL = \bar{\bar{X}} - A_2 \cdot \bar{R}$

$= 16,27 - 0,577 \cdot 0,47 = 15,99$

All observations are within control limits, the process exhibit statistical control.

For R chart; $\bar{R} = 0,47$

Table Values for $n=5$; $D_4 = 2,114$

$D_3 = 0$

$UCL = D_4 \bar{R}$

$= 2,114 \cdot 0,47 = 1,004$

$LCL = D_3 \bar{R}$

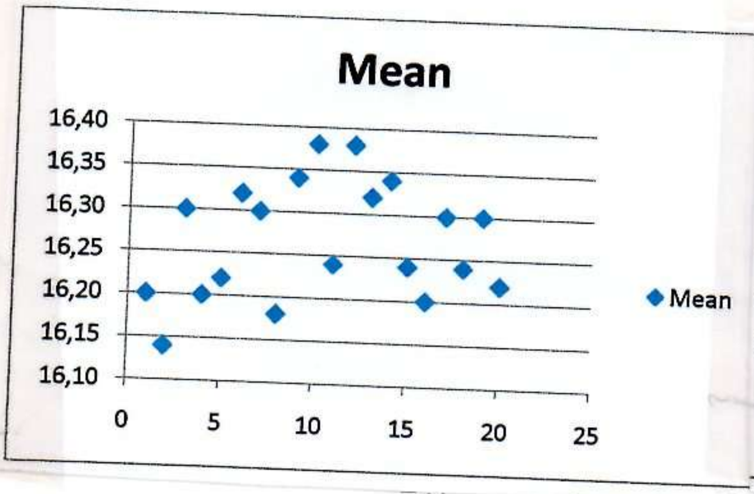
$= 0 \cdot 0,47 = 0$

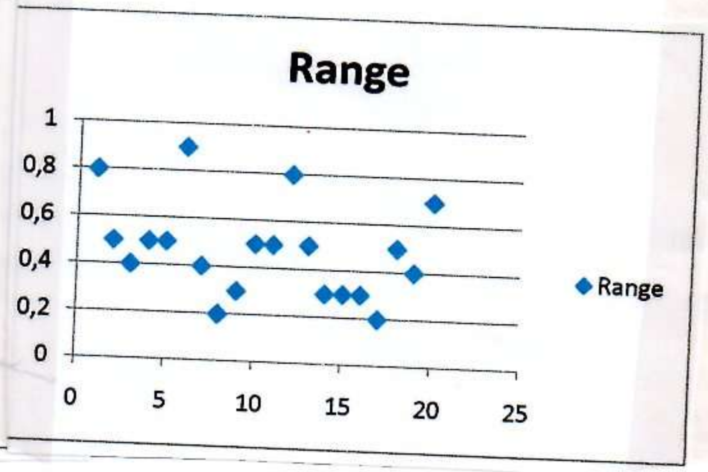
All observations are within control limits. The process exhibit statistical control.

b) Estimated Process mean $= \hat{\mu} = \bar{\bar{X}} = 16,27$

Estimated Process standard Deviation $= \dfrac{\bar{R}}{d_2} = \dfrac{0,47}{2,326} = 0,202 = \hat{\sigma}$

Sample No.	(I) Total(x_i)	(II) Max(x_i)	(III) Min(x_i)	(I)/5 Mean	(II)-(III) Range	IV Total(x_i^2)	$\frac{(IV)-\frac{(I)^2}{5}}{4}$ s^2	$\sqrt{s^2}$ s
1	81,0	16,6	15,8	16,20	0,8	1312,54	0,08	0,29
2	80,7	16,4	15,9	16,14	0,5	1302,71	0,05	0,23
3	81,5	16,5	16,1	16,30	0,4	1328,55	0,03	0,16
4	81,0	16,4	15,9	16,20	0,5	1312,34	0,04	0,19
5	81,1	16,5	16,0	16,22	0,5	1315,63	0,05	0,22
6	81,6	16,7	15,8	16,32	0,9	1332,26	0,14	0,37
7	81,5	16,5	16,1	16,30	0,4	1328,61	0,04	0,20
8	80,9	16,3	16,1	16,18	0,2	1308,99	0,01	0,08
9	81,7	16,5	16,2	16,34	0,3	1335,03	0,01	0,11
10	81,9	16,6	16,1	16,38	0,5	1341,67	0,04	0,19
11	81,2	16,4	15,9	16,24	0,5	1318,86	0,04	0,21
12	81,9	16,7	15,9	16,38	0,8	1341,95	0,11	0,33
13	81,6	16,6	16,1	16,32	0,5	1331,9	0,05	0,22
14	81,7	16,5	16,2	16,34	0,3	1335,03	0,01	0,11
15	81,2	16,4	16,1	16,24	0,3	1318,74	0,01	0,11
16	81,0	16,3	16,0	16,20	0,3	1312,26	0,02	0,12
17	81,5	16,4	16,2	16,30	0,2	1328,49	0,01	0,10
18	81,2	16,5	16,0	16,24	0,5	1318,86	0,04	0,21
19	81,5	16,4	16,0	16,30	0,4	1328,57	0,03	0,17
20	81,1	16,5	15,8	16,22	0,7	1315,81	0,09	0,30
			TOTAL =	325,36	9,50	TOTAL =	0,89	3,93
			MEAN =	16,27	0,47	MEAN =	0,04	0,20





(d) Process Capability

* A process aims to produce maximum percentage of output that are within "Specification Limits". Note that, there is NO mathematical or statistical relationship between the control limits and specification limits.

Specification limits are set by management;

$USL = \mu + \varepsilon$; $LSL = \mu - \varepsilon$.

Control limits are driven by the natural variability of the process; that is, by the natural tolerance limits of the process;

$UNTL = \bar{\bar{X}} + 3.\hat{\sigma}$; $LNTL = \bar{\bar{X}} - 3.\hat{\sigma}$

* Process Capability measures what percentage of the output is within specification limits. $\bar{\bar{X}} = 16{,}27$; $\hat{\sigma} = 0{,}202$;

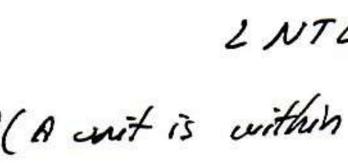
we have; $UNTL = \bar{\bar{X}} + 3\hat{\sigma} = 16{,}27 + 3 . 0{,}202 = 16{,}876$

$LNTL = \bar{\bar{X}} - 3.\hat{\sigma} = 16{,}27 - 3 . 0{,}202 = 15{,}664$

P(A unit is within Specification limits)

$= P(16{,}2 - 0{,}5 < X < 16{,}2 + 0{,}5) = P(15{,}7 < X < 16{,}7)$

$= P\left(\frac{15{,}7 - 16{,}27}{0{,}202} < \frac{X - \hat{\mu}}{\hat{\sigma}} < \frac{16{,}7 - 16{,}27}{0{,}202}\right) = P(-2{,}82 < Z < 2{,}13)$



$= 0{,}98341 - (1 - 0{,}9976) = 0{,}981$

And, P(Outside Specification limits) $= 1 - 0{,}981 = 0{,}019$

which may be considered as a "Bad Performance"!
1,9% may seem to be low but considering the whole production, 1900 units out of one million (parts per million, ppm) are nonconforting, which is too much costly for the producer.

Note that, the process is in control, but does not perform well. The process produces items within (15, ~~664~~; 16, 876), 99,73% of the time. However, this does NOT ~~...~~ result a good fit with specification limits 16,2 ± 0,5

e) $P(X < LSL) = P(X < 15,7) = P(Z < -2,82) = 1 - 0,9976 = 0,0024$

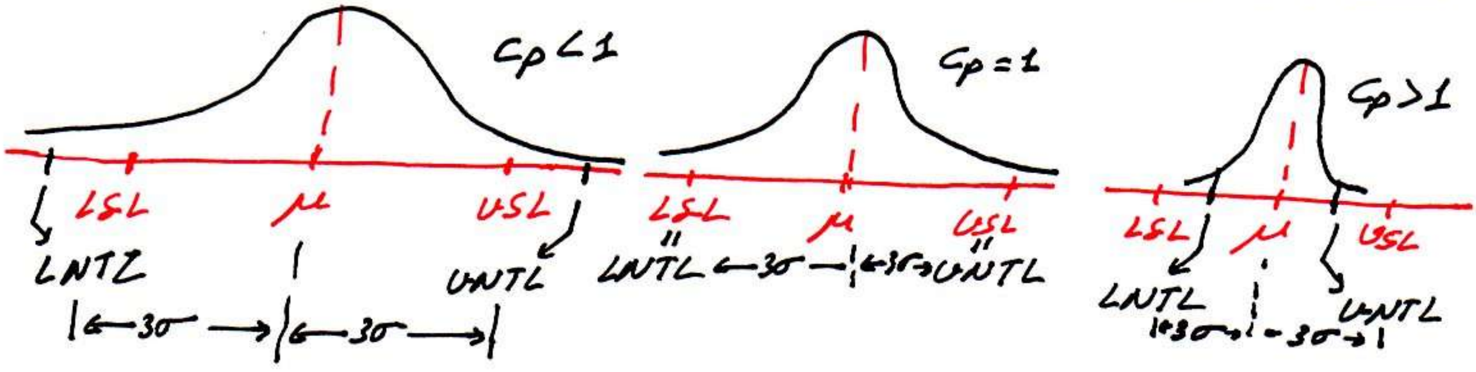
Also, $P(X > USL) = P(X > 16,7) = P(Z > 2,13) = 1 - 0,9834 = 0,0166$

0,24% of the products are lower than LSL, and 1,66% of the products are greater than USL. So, 1,66% + 0,24% = 1,9% of the products are outside the specification limits, as we found in part (d).

Process Capability Ratio (PCR): C_p

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

we have:
$$\begin{cases} C_p > 1 \Rightarrow \text{Process uses less than 100\% of the tolerance band} \\ C_p = 1 \Rightarrow \text{Process uses 100\% of the tolerance band} \\ C_p < 1 \Rightarrow \text{Process uses more than 100\% of the tolerance band} \end{cases}$$



Note that, here we assume the process is centered at $\bar{\bar{X}} = \mu = \frac{USL + LSL}{2}$. Given the specification limits and variance, this value of the mean give the minimum possible nonforting items proportion.

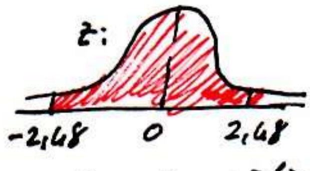
Also note that, the percentage of the specification band that the process uses up is given by;

$$P = \left(\frac{1}{C_p}\right) \cdot 100\%$$

f) If the process mean were centered at $\mu = 16{,}2$; what would be the effect on nonconforting items percentage?

$$P(\text{within Specification limits}) = P(15{,}7 < X < 16{,}7)$$

$$= P\left(\frac{15{,}7 - 16{,}2}{0{,}202} < \frac{X-\mu}{\hat{\sigma}} < \frac{16{,}7 - 16{,}2}{0{,}202}\right) = P(-2{,}48 < Z < 2{,}48)$$



$$= 0{,}9934 - (1 - 0{,}9934) = 0{,}9868$$

$$P(\text{Outside Specification limits}) = 1 - 0{,}9838 = 0{,}0132$$

When the process mean is 16,27, this probability was 0,0190. It has decreased, but still is NOT satisfactory. The process standard deviation should also decrease.

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g) what percentage at the specification band is used up by the process?

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{16,7 - 15,7}{6 \cdot 0,202} = 0,825$$

$$P = \frac{1}{0,825} \cdot 100 = 121,2\%$$

The specification band is exceeded by 21,2%.

6.8. Samples of $n = 6$ items each are taken from a process at regular intervals. A quality characteristic is measured, and $\bar{x}$ and R values are calculated for each sample. After 50 samples, we have

$$\sum_{i=1}^{50} \bar{x}_i = 2000 \quad \text{and} \quad \sum_{i=1}^{50} R_i = 200$$

Assume that the quality characteristic is normally distributed.

(a) Compute control limits for the $\bar{x}$ and R control charts.

(b) All points on both control charts fall between the control limits computed in part (a). What are the natural tolerance limits of the process?

(c) If the specification limits are 41 ± 5.0, what are your conclusions regarding the ability of the process to produce items within these specifications?

(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process producing?

6.8) a) $\bar{\bar{X}} = \frac{2000}{50} = 40$

$\bar{R} = \frac{200}{50} = 4$

A_2 for $n=6$ = 0,483

$D_3 = 0$; $D_4 = 2,004$

$\bar{X}$-Chart

$UCL = 40 + 0,483 \cdot 4 = 41,932$

central line $= \bar{\bar{X}} = 40$

$LCL = 40 - 0,483 \cdot 4 = 38,068$

R-Chart

$UCL = 2,004 \cdot 4 = 8,016$

central line $= \bar{R} = 4$

$LCL = 0 \cdot 4 = 0$

b) $d_2 = 2,534$

$\hat{\sigma} = \frac{4}{2,534} = 1,58$

$UNTL = \bar{\bar{X}} + 3\hat{\sigma} = 40 + 3 \cdot 1,58 = 43,54$

$LNTL = \bar{\bar{X}} - 3\hat{\sigma} = 40 - 3 \cdot 1,58 = 36,46$

c) $\hat{C}_p = \frac{46 - 36}{6 \cdot 1,18} = 1,41$; $P = \frac{1}{1,41} \cdot 100 = 70,8\%$

Process uses 70,8% of the specification band.

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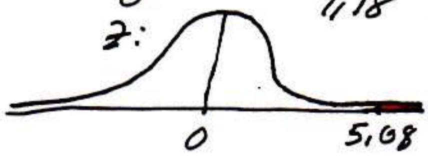
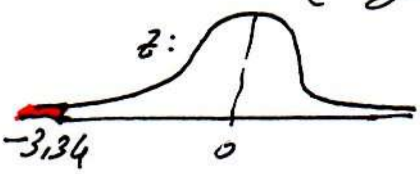
d) $P(X>46) = P\left(\frac{X-\hat{\mu}}{\hat{\sigma}} > \frac{46-40}{1{,}18}\right) = P(Z>5{,}08)$
$= 0{,}00000$



Almost NONE of the parts is Reworked.

$P(X<36) = P\left(\frac{X-\hat{\mu}}{\hat{\sigma}} < \frac{36-40}{1{,}18}\right) = P(Z<-3{,}34)$

$= 1 - 0{,}99958 = 0{,}00042$



0,042% of the parts are scrapped.

Changing Sample Size on $\bar{X}$ and R charts;

If the sample size of the process changes, the new confidence limits are found by the following formulas:

$\bar{R}_{old}$ = Average range for the old sample size

$\bar{R}_{new}$ = Average range for the new sample size

n_{old} = Old Sample Size

n_{new} = new Sample Size

$d_2(old)$ = factor d_2 for the old sample size

$d_2(new)$ = factor d_2 for the new sample size

For $\bar{X}$ chart, new Control limits are;

$UCL = \bar{\bar{X}} + A_2\left[\frac{d_2(new)}{d_2(old)}\right] \cdot \bar{R}_{old}$; $LCL = \bar{\bar{X}} - A_2\left[\frac{d_2(new)}{d_2(old)}\right] \cdot \bar{R}_{old}$

For R chart, the new parameters are;

$$UCL = D_4 \cdot \left[\frac{d_2\,(new)}{d_2\,(old)}\right] \bar{R}_{old} \; ; \; CL = \bar{R}_{new} = \left[\frac{d_2\,(new)}{d_2\,(old)}\right] \cdot \bar{R}_{old} \text{ and}$$

$$LCL = \max\left\{0,\; D_3 \cdot \left[\frac{d_2\,(new)}{d_2\,(old)}\right] \bar{R}_{old}\right\}$$

6.12. Consider the $\bar{x}$ and R charts you established in Exercise 6.2 using $n = 5$.

(a) Suppose that you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of $n = 3$. What limits would be used on the $\bar{x}$ and R charts?

(b) What would be the impact of the decision you made in part (a) on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

6.12) for $n=5$; $d_2\,(old) = 2{,}326$

for $n=3$; $d_2\,(new) = 1{,}693$

$A_2 = 1{,}023$ (we use A_2 for new sample size, $n=3$)

$\bar{\bar{X}} = 16{,}27$; $\bar{R}_{old} = 0{,}47$; $D_4 = 2{,}574$

$D_3 = 0$

$\hat{\sigma} = 0{,}202$

a) For $\bar{X}$ chart, New Control limits Are;

$$UCL = 16{,}27 + 1{,}023 \cdot \left[\frac{1{,}693}{2{,}326}\right] = 17{,}01$$

$$LCL = 16{,}27 - 1{,}023 \cdot \left[\frac{1{,}693}{2{,}326}\right] = 15{,}53$$

For R chart, New Control limits Are;

$$UCL = 2{,}574 \cdot \left[\frac{1{,}693}{2{,}326}\right] \cdot 0{,}47 = 0{,}88 \; ; \; \bar{R}_{new} = \left[\frac{1{,}693}{2{,}326}\right] 0{,}47 = 0{,}34$$

$$LCL = \max\{0,\, 0\} = 0$$

b) Old limits for $\bar{X}$ Chart: $UCL = 16{,}56$; $LCL = 15{,}99$; $\hat{\sigma} = 0{,}202$

New limits for $\bar{X}$ Chart: $UCL = 17{,}01$; $LCL = 15{,}53$; $\hat{\sigma} = 0{,}202$

Note that, $\hat{\sigma}$ does not change ($d_2\,(new)$ cancels out)

Let, mean is increased by 2σ and became

$$\bar{\bar{X}} + 2\hat{\sigma} = 16{,}27 + 2 \cdot 0{,}202 = 16{,}674$$

with old Limits;

$$P(\text{Detect}) = 1 - P(15{,}99 < \bar{x} < 16{,}54 \, ; \, \hat{\mu} = 16{,}674)$$

$$= 1 - P\left(\frac{15{,}99 - 16{,}674}{0{,}202/\sqrt{5}} < \frac{\bar{x} - \hat{\mu}}{\hat{\sigma}/\sqrt{n}} < \frac{16{,}54 - 16{,}674}{0{,}202/\sqrt{5}} \right)$$

$$= 1 - P(-7{,}57 < Z < -1{,}48) = 1 - (1 - 0{,}9306) = 0{,}9306.$$

with New Limits;

$$P(\text{Detect}) = 1 - P\left(\frac{15{,}53 - 16{,}674}{0{,}202/\sqrt{3}} < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{17{,}01 - 16{,}674}{0{,}202/\sqrt{3}} \right)$$

$$= 1 - P(-9{,}81 < Z < 1{,}15) = 1 - 0{,}8749 = 0{,}1251.$$

Detection probability significantly decreases because

(i) Confidence limits get wider. Probability of being within confidence limits becomes more probable although there is a shift.

(ii) Less information about the population makes it harder to detect a shift.

Charts Based on Standard Values;

Let; μ and σ of the process is known. Then the control limits for $\bar{x}$ and R charts are;

$\bar{x}$-Chart

$UCL = \mu + 3 \cdot \frac{\sigma}{\sqrt{n}} = \mu + A\sigma$

Central line $= \mu$

$LCL = \mu - 3 \cdot \frac{\sigma}{\sqrt{n}} = \mu - A\sigma$

R-chart;

$UCL = d_2\sigma + 3d_3\sigma = D_2\sigma$

Central Line $= d_2\sigma$

$LCL = d_2\sigma - 3d_3\sigma = D_1\sigma$

6.20. A process is to be monitored with standard values $\mu = 10$ and $\sigma = 2.5$. The sample size is $n = 2$.
(a) Find the center line and control limits for the $\overline{x}$ chart.
(b) Find the center line and control limits for the R chart.

6.20) $\mu = 10$; $\sigma = 2,5$; $n = 2$

$A = 2,121$; $D_2 = 3,686$; $D_1 = 0$; $d_2 = 1,128$

a) $\overline{x}$ - Chart ;

$UCL = 10 + 2,121 \cdot 2,5 = 15,3$

Central line $= 10$

$LCL = 10 - 2,121 \cdot 2,5 = 4,7$

b) R - Chart

$UCL = 3,686 \cdot 2,5 = 9,22$

Central Line $= 1,128 \cdot 2,5 = 2,82$

$LCL = 0$

Average Run Length

Let the mean of the process is centered at $\mu = \mu_0$ but somehow, it has shifted to $\mu = \mu_1$. Remember, taking each sample is a hypothesis test

$H_0 : \mu = \mu_0$

$H_A : \mu \neq \mu_0$

$\alpha = 0,0027$ (Because of 3σ limits)

And we Reject H_0 if $\overline{X}$ is out of Control Limits. Remember, Type-II Error $= \beta = P($Do NOT Reject $H_0 ; H_A)$

Then; $\beta = P($Do NOT Reject $H_0 ; \mu = \mu_1)$

which means NOT detecting the shift in the process.

Let, N: # of samples taken to detect the shift first time. Then;

$N \sim \text{Geometric}(1-\beta)$

Because $1-\beta = P($Detecting the shift.

Remember; $X \sim Geometric(p) \Rightarrow E(X) = \frac{1}{p}$.

Average run length is the expected Number of samples to take to detect the mean shift. Then, when the process is out of control for $\mu = \mu_1$;

$$ARL_1 = \frac{1}{1-\beta}$$

If the process is in control,

α = Type I Error = Probability of taking an out of control signal

$$\alpha = P(\text{Reject } H_0 ; H_0)$$

which is called ~~an~~ false alarm.

Then, the expected Number of samples that give a false alarm when the process is in control $\mu = \mu_0$ is;

$$ARL_0 = \frac{1}{\alpha}$$

6.43. $\bar{x}$ and R charts with $n = 4$ are used to monitor a normally distributed quality characteristic. The control chart parameters are

$\bar{x}$ Chart	R Chart
UCL = 815	UCL = 46.98
Center line = 800	Center line = 20.59
LCL = 785	LCL = 0

Both charts exhibit control. What is the probability that a shift in the process mean to 790 will be detected on the first sample following the shift?

6.44. Consider the $\bar{x}$ chart in Exercise 6.43. Find the average run length for the chart.

6.43) We have; $\bar{R} = 20{,}59$; $d_2 = 2{,}059$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{20{,}59}{2{,}59} = 7{,}95$$

$$\beta = P(785 < \bar{X} < 815 ; \mu_1 = 790)$$

$$= P\left(\frac{785-790}{7{,}95/\sqrt{4}} < \frac{\bar{X}-\mu}{\hat{\sigma}/\sqrt{n}} < \frac{815-790}{7{,}95/\sqrt{4}}\right)$$

$$= P(-1{,}26 < Z < 6{,}29)$$

$$= 0{,}8962$$

$$1-\beta = P(\text{Detect}) = 1 - 0{,}8962 = 0{,}1038$$

6.44) $ARL_1 = \frac{1}{1-\beta} = \frac{1}{0{,}1038} = 9{,}63$

6.41. An $\bar{x}$ chart with three-sigma limits has parameters as follows:

$$UCL = 104$$
$$\text{Center line} = 100$$
$$LCL = 96$$
$$n = 5$$

Suppose the process quality characteristic being controlled is normally distributed with a true mean of 98 and a standard deviation of 8. What is the probability that the control chart would exhibit lack of control by at least the third point plotted?

6.42. Consider the $\bar{x}$ chart defined in Exercise 6.41. Find the ARL_1 for the chart.

6.41) $\mu_1 = 98$; $\sigma = 8$; $n = 5$

$\beta = P(96 < \bar{x} < 104 \,;\, \mu_1 = 98)$

$= P\left(\frac{96-98}{8/\sqrt{5}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{104-98}{8/\sqrt{5}}\right)$

$= P(-0{,}56 < z < 1{,}68)$

$= 0{,}9535 - (1 - 0{,}7123) = 0{,}6658$

$1 - \beta = 1 - 0{,}6658 = 0{,}3342$

Let N: # of samples to detect the shift.

$N \sim \text{Geometric}(p = 0{,}3342)$

$f(n) = (1 - 0{,}3342)^{n-1} \cdot 0{,}3342$

$P(N \leq 3) = f(1) + f(2) + f(3) = 0{,}3342 + 0{,}6658 \cdot 0{,}3342 + 0{,}6658^2 \cdot 0{,}3342$

$= 0{,}7049$

6.42) $ARL_1 = \frac{1}{1-\beta} = \frac{1}{0{,}3342} = 2{,}99$

* Note that, if the process is in control and 3σ limits are used, then,

$\alpha = P(-3 < z < 3) = 2 \cdot (1 - 0{,}99865) = 0{,}0027$

And, $ARL_0 = \frac{1}{0{,}0027} = 370{,}4$

* Sometimes, probability limits are used instead of $k.\sigma$ control limits. Remember, $(1-\alpha).100\%$ C.I. for μ was; $\bar{X} \pm z_{\alpha/2} . \frac{\sigma}{\sqrt{n}}$. Then, to find control limits, simply change k by $z_{\alpha/2}$.

6.35. An $\bar{x}$ chart on a normally distributed quality characteristic is to be established with the standard values $\mu = 100$, $\sigma = 8$, and $n = 4$. Find the following:
(a) The two-sigma control limits.
(b) The 0.005 probability limits.

6.35) $\mu = 100; \sigma = 8; n = 4$

a) 2σ control limits are;

$$UCL = 100 + 2 . \frac{8}{\sqrt{4}} = 108$$

$$LCL = 100 - 2 . \frac{8}{\sqrt{4}} = 92$$

b)

$$UCL = 100 + 2,81 . \frac{8}{\sqrt{4}} = 111,24$$

$$LCL = 100 - 2,81 . \frac{8}{\sqrt{4}} = 88,76$$

Control Charts for $\bar{X}$ & S

$$S_i^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} \quad ; \quad S = \sqrt{S^2} \quad ; \quad \bar{S} = \frac{\sum S_i}{m}$$

* Control limits for S charts are;

if σ is known;

$$UCL = c_4 \sigma + 3\sigma \sqrt{1-c_4} = B_6 \sigma$$

$$\text{Central line} = c_4 \sigma$$

$$LCL = c_4 \sigma - 3\sigma \sqrt{1-c_4} = B_5 \sigma$$

if σ is unknown;

$$UCL = \bar{S} + 3 \frac{\bar{S}}{c_4} \sqrt{1-c_4} = B_4 \bar{S}$$

$$\text{Center line} = \bar{S}$$

$$LCL = \bar{S} - 3 . \frac{\bar{S}}{c_4} \sqrt{1-c_4} = B_3 \bar{S} \quad \text{and} \quad \hat{\sigma} = \frac{\bar{S}}{c_4}$$

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* Control limits for $\overline{X}$ chart are

$$UCL = \overline{\overline{X}} + \frac{3\bar{s}}{c_4\sqrt{n}} = \overline{\overline{X}} + A_3\bar{s}$$

$$\text{Central line} = \overline{\overline{X}}$$

$$LCL = \overline{\overline{X}} - \frac{3\bar{s}}{c_4\sqrt{n}} = \overline{\overline{X}} - A_3\bar{s}$$

* The s^2 Control Chart Control Limits are; (with probability limits)

$$UCL = \frac{\bar{s}^2}{n-1} \chi^2_{\alpha/2;\,n-1}$$

$$\text{Center Line} = \bar{s}^2$$

$$LCL = \frac{\bar{s}^2}{n-1} \cdot \chi^2_{1-\alpha/2;\,n-1}$$

Ex: Going back to our initial Exercise 6.2, we have the following results using s chart.

For $n=5$; $B_3=0$; $B_4=2{,}089$; $c_4=0{,}94$; $A_3=1{,}427$

The output statistics are; $\overline{\overline{X}}=16{,}27$; $\bar{s}=0{,}20$; $\bar{s}^2=0{,}04$

a) Control limits for s chart are;

$UCL = B_4\bar{s} = 2{,}089 \cdot 0{,}20 = 0{,}412$

Central line $= \bar{s} = 0{,}20$

$LCL = 0$

* Note that there is a very small difference for $\bar{x}$ chart.

Control limits for $\overline{X}$ chart are;

$UCL = \overline{\overline{X}} + A_3\bar{s} = 16{,}27 + 1{,}427 \cdot 0{,}20 = 16{,}56$

Central line $= \overline{\overline{X}} = 16{,}27$

$LCL = \overline{\overline{X}} - A_3 \cdot \bar{s} = 16{,}27 - 1{,}427 \cdot 0{,}20 = 15{,}98$

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b) Estimated Process Mean $= \hat{\mu} = \bar{\bar{x}} = 16,27$

Estimated Process Standard Deviation $= \hat{\sigma} = \dfrac{\bar{s}}{c_4} = \dfrac{0,20}{0,94} = 0,213$

d-e) Since the control limits for $\bar{x}$ chart and estimated standard deviation are quite close, the results will be almost the same with that of the R-chart.

f) The 0,01 probability limits for s^2 Control Chart are;

df=4

$\chi^2_{0,995} = 0,21$ $\qquad$ $16,86 = \chi^2_{0,005}$

$$UCL = \frac{\bar{s}^2}{n-1} \cdot \chi^2_{0,005} = \frac{0,04}{4} \cdot 16,86 = 0,17$$

$$\text{Central Line} = \bar{s}^2 = 0,04$$

$$LCL = \frac{\bar{s}^2}{n-1} \cdot \chi^2_{0,995} = \frac{0,04}{4} \cdot 0,21 = 0,0002$$

6.17. Samples of size $n = 5$ are collected from a process every half hour. After 50 samples have been collected, we calculate $\bar{\bar{x}} = 20.0$ and $\bar{s} = 1.5$. Assume that both charts exhibit control and that the quality characteristic is normally distributed.

(a) Estimate the process standard deviation.
(b) Find the control limits on the $\bar{x}$ and s charts.
(c) If the process mean shifts to 22, what is the probability of concluding that the process is still in control?

6.17) $n = 5;\ m = 50;\ c_4 = 0,94$

a) $\hat{\sigma} = \dfrac{1,5}{0,94} = 1,596$

b) $\bar{x}$ chart

$UCL = 20 + 1,427 \cdot 1,5 = 22,14$

Central line $= 20,0$

$LCL = 20 - 1,427 \cdot 1,5 = 17,86$

s chart

$UCL = 2,089 \cdot 1,5 = 3,134$

Central line $= \bar{s} = 1,5$

$LCL = 0$

c) $\beta = P(17,86 < \bar{x} < 20,14;\ \mu_1 = 22)$

$$= P\left(\frac{17,86-22}{1,596/\sqrt{5}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{22,14-22}{1,596/\sqrt{5}}\right) = P(-5,8 < z < 0,20)$$

$$= 0,5793$$

6.9. Control charts on $\bar{x}$ and s are to be maintained on the torque readings of a bearing used in a wingflap actuator assembly. Samples of size $n = 10$ are to be used, and we know from past experience that when the process is in control, bearing torque has a normal distribution with mean $\mu = 80$ inch-pounds and standard deviation $\sigma = 10$ inch-pounds. Find the center line and control limits for these control charts.

6.9) $n=10;\ \mu=80;\ \sigma=10$

x-chart

$UCL = \mu + A\sigma = 80 + 0{,}949 \cdot 10 = 89{,}49$

Central Line $= \mu = 80$

$LCL = \mu - A\sigma = 80 - 0{,}949 \cdot 10 = 70{,}51$

s-Chart (σ known)

$UCL = B_6\,\sigma = 1{,}669 \cdot 10 = 16{,}69$

Central Line $= c_4\,\sigma = 0{,}9727 \cdot 10 = 9{,}73$

$LCL = B_5\,\sigma = 0{,}276 \cdot 10 = 2{,}76$

6.15. Samples of $n = 4$ items are taken from a process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and s values are calculated for each sample. After 50 subgroups have been analyzed, we have

$$\sum_{i=1}^{50} \bar{x}_i = 1000 \quad \text{and} \quad \sum_{i=1}^{50} s_i = 72$$

(a) Compute the control limit for the $\bar{x}$ and s control charts.
(b) Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process?
(c) If the specification limits are 19 ± 4.0, what are your conclusions regarding the ability of the process to produce items conforming to specifications?
(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process now producing?
(e) If the process were centered at $\mu = 19.0$, what would be the effect on percent scrap and rework?

6.15) a) $\bar{\bar{x}} = \frac{1000}{50} = 20$

$\bar{s} = \frac{72}{50} = 1{,}44$ $\qquad c_4 = 0{,}9213$

$n=4 \Rightarrow B_3 = 0;\ B_4 = 2{,}226;\ A_3 = 1{,}628$

$\bar{x}$-chart;

$UCL = 20 + 1{,}628 \cdot 1{,}44 = 22{,}344$

Central Line $= 20$

$LCL = 20 - 1{,}628 \cdot 1{,}44 = 17{,}656$

s-chart

$UCL = 2{,}226 \cdot 1{,}44 = 3{,}205$

Central Line $= 1{,}44$

$LCL = 0$

b) $\hat{\sigma} = \frac{1{,}44}{0{,}9213} = 1{,}563$ $\qquad UNTL = 20 + 3 \cdot 1{,}563 = 24{,}689$

$LNTL = 20 - 3 \cdot 1{,}563 = 15{,}311$

c) $\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{23 - 15}{6 \cdot 1{,}563} = 0{,}853$; $P = 100 \cdot \frac{1}{0{,}853} = 117{,}23\%$

The tolerance band of the specification is exceeded by 17,23%.

d) $P(\text{Rewoke}) = P(X > 23) = P\left(\frac{X-\hat{\mu}}{\hat{\sigma}} > \frac{23-20}{1,563}\right) = P(Z > 1,92)$

$= 1 - 0,9726 = 0,0274$

$P(\text{Scrap}) = P(X < 15) = P\left(\frac{X-\mu}{\hat{\sigma}} < \frac{15-20}{1,563}\right) = P(Z < -3,20)$

$= 1 - 0,9993 = 0,0007$

P(Out of Tolerance Limits) = $0,0274 + 0,0007 = 0,0281$

e) $P(\text{Rewoke}) = P(\text{Scrap}) = P(X > 23) = P\left(\frac{X-\hat{\mu}}{\hat{\sigma}} > \frac{23-19}{1,563}\right)$

$= P(Z > 2,56) = 1 - 0,9948 = 0,0052$

Rewoke Proportion will increase considergbly, Scrap Proportion will increase slightly

P(Out of Tolerance Limits $= 2 \cdot 0,0052 = 0,0104 \rightarrow$ min. given $\hat{\sigma}$.

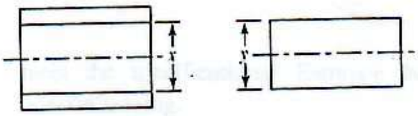

FIGURE 6.28 Parts for Exercise 6.34.

6.34. Two parts are assembled as shown in Figure 6.28. Assume that the dimensions x and y are normally distributed with means μ_x and μ_y and standard deviations σ_x and σ_y, respectively. The parts are produced on different machines and are assembled at random. Control charts are maintained on each dimension for the range of each sample ($n = 5$). Both range charts are in control.

(a) Given that for 20 samples on the range chart controlling x and 10 samples on the range chart controlling y, we have

$$\sum_{i=1}^{20} R_{x_i} = 18.608 \quad \text{and} \quad \sum_{i=1}^{10} R_{y_i} = 6.978$$

Estimate σ_x and σ_y.

(b) If it is desired that the probability of a smaller clearance (i.e., $x - y$) than 0.09 should be 0.006, what distance between the average dimensions (i.e., $\mu_x - \mu_y$) should be specified?

6.34) a) $n = 10 \Rightarrow d_2 = 3,078$

$n = 20 \Rightarrow d_2 = 3,735$

$\bar{R}_x = \frac{18,608}{20} = 0,93$

$\bar{R}_y = \frac{6,978}{10} = 0,70$

$\hat{\sigma}_x = \frac{\bar{R}_x}{d_{2(20)}} = \frac{0,93}{3,735} = 0,249$

$\hat{\sigma}_y = \frac{\bar{R}_y}{d_{2(20)}} = \frac{0,70}{3,735} = 0,187$

b) $C = X - Y$

$\mu_c = E(C) = E(X-Y) = E(X) - E(Y) = \mu_x - \mu_y$

$\sigma_c^2 = Var(C) = Var(X-Y) = Var(X) + Var(Y)$

$\hat{\sigma}_c^2 = 0,249^2 + 0,187^2 = 0,311^2$

$\hat{\sigma}_c = 0,311$

$P(C \leq 0,09) = 0,006$

$P\left(\frac{C-\mu_c}{\hat{\sigma}_c} \leq \frac{0,09-\mu_c}{0,311}\right) = 0,006$

$P\left(Z \leq \underbrace{\frac{0,09-\mu_c}{0,311}}_{=-2,51}\right) = 0,006$

$\frac{0,09-\mu_c}{0,311} = -2,51$

$\mu_c = 0,09 + 0,311 \cdot 2,51$

$\mu_c = 0,87$

Individual Measurements

When, only single observation for each period is available, we use MR and $\bar{X}$ control charts using individual observations. Note that; $MR_i = |x_i - x_{i-1}|$

$\bar{X}$-Chart

$UCL = \bar{X} + 3 \cdot \frac{\overline{MR}}{d_2}$

Central line $= \bar{X}$

$LCL = \bar{X} - 3 \cdot \frac{\overline{MR}}{d_2}$

MR-chart

$UCL = D_4 \cdot \overline{MR}$

Central line $= \overline{MR}$

$LCL = D_3 \cdot \overline{MR}$

where d_2, D_3 and D_4 values are for $n=2$.

Estimated mean of the process $= \hat{\mu} = \bar{X}$

Estimated std-dev. of the process $= \hat{\sigma} = \frac{\overline{MR}}{d_2}$

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Car no.	x_i	MR_i
1	16,11	
2	16,08	0,03
3	16,12	0,04
4	16,10	0,02
5	16,10	0,00
6	16,11	0,01
7	16,12	0,01
8	16,09	0,03
9	16,12	0,03
10	16,10	0,02
11	16,09	0,01
12	16,07	0,02
13	16,13	0,06
14	16,12	0,01
15	16,10	0,02
16	16,08	0,02
17	16,13	0,05
18	16,15	0,02
19	16,12	0,03
20	16,10	0,02
21	16,08	0,02
22	16,07	0,01
23	16,11	0,04
24	16,13	0,02
25	16,10	0,03
TOTAL	402,63	0,57
MEAN	16,1052	0,02375

6.45. One-pound coffee cans are filled by a machine, sealed, and then weighed automatically. After adjusting for the weight of the can, any package that weighs less than 16 oz is cut out of the conveyor. The weights of 25 successive cans are shown in Table 6E.14. Set up a moving range control chart and a control chart for individuals. Estimate the mean and standard deviation of the amount of coffee packed in each can. Is it reasonable to assume that can weight is normally distributed? If the process remains in control at this level, what percentage of cans will be underfilled?

TABLE 6E.14
Can Weight Data for Exercise 6.45

Can Number	Weight	Can Number	Weight
1	16.11	14	16.12
2	16.08	15	16.10
3	16.12	16	16.08
4	16.10	17	16.13
5	16.10	18	16.15
6	16.11	19	16.12
7	16.12	20	16.10
8	16.09	21	16.08
9	16.12	22	16.07
10	16.10	23	16.11
11	16.09	24	16.13
12	16.07	25	16.10
13	16.13		

For $n=2$; $d_2 = 1{,}128$; $D_3 = 0$; $D_4 = 3{,}267$

6.45) we have; $\bar{X} = 16{,}105$; $\overline{MR} = 0{,}0238$

$\bar{X}$ - Chart

$UCL = 16{,}105 + 3 \cdot \frac{0{,}0238}{1{,}128} = 16{,}168$

Central Line $= 16{,}105$

$LCL = 16{,}105 - 3 \cdot \frac{0{,}0238}{1{,}128} = 16{,}042$

MR - Chart

$UCL = 3{,}267 \cdot 0{,}0238 = 0{,}0778$

Central Line $= 0{,}0238 =$

$LCL = 0$

Estimated process mean $= \hat{\mu} = 16{,}105$

Estimated process std-dev $= \hat{\sigma} = \frac{0{,}0238}{1{,}128} = 0{,}0211$