STATISTICS for Lawyers & Social Sciences | CHAPTERS 4 & 5

Basic Principle of Counting

Let, we want to select one element from a set A_1, 1 element from a set A_2, ..., one element from a set A_k. Also let the number of elements in the sets are:

$$n(A_1) = n_1 \ , \ n(A_2) = n_2 , \ ---- , \ n(A_k) = n_k .$$

The whole selection can be made in

$$n_1 \cdot n_2 \cdot \ldots \cdot n_k \text{ different ways.}$$

Example: A menu consists of one soup, one main meal and one desert. If there are 3 soups, 5 main meals and 2 deserts, in how many different ways one can construct a menu? Show the tree diagram.

Answer; Let,
$$S = \{s_1, s_2, s_3\}$$
$$M = \{m_1, m_2, m_3, m_4, m_5\}$$
$$D = \{d_1, d_2\}$$

We will choose one element from each set and construct a menu: (s_i, m_j, d_k) where $i \in \{1,2,3\}$, $j \in \{1,2,3,4,5\}$, $k \in \{1,2\}$

Number of selections is;

$$\underline{3} \cdot \underline{5} \cdot \underline{2} = 30$$

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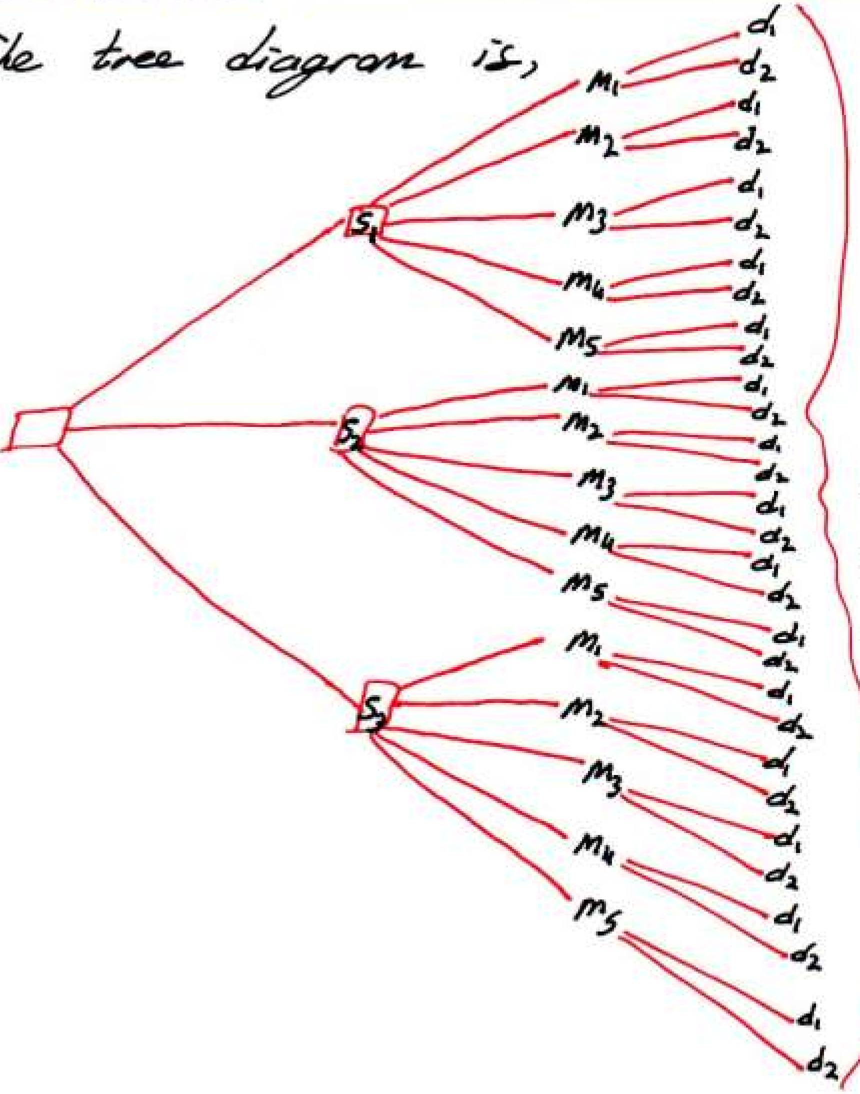
The tree diagram is,



Each way is a different menu, there are 30 different menus.

4.10 A furniture store sells living room chairs which are available in 5 styles, 10 types of fabrics, and 8 colors. In how many ways can a customer buy a living room chair?

4.10) $\underline{5} \cdot \underline{10} \cdot \underline{8} = 400$

$n(S) = 5$

$n(F) = 10$

$n(C) = 8$

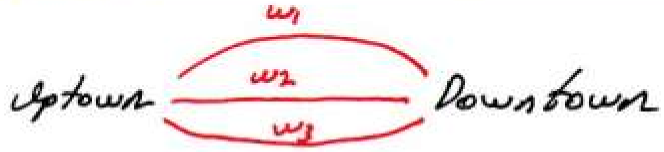
4.14 Given that there are three routes by which a truck can travel from uptown to downtown, in how many different ways can a driver plan a trip from uptown to downtown and back if

(a) the driver must travel both ways by the same route;

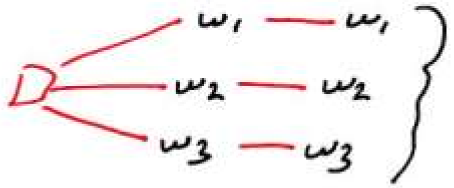
(b) the driver can, but need not, travel both ways by the same route;

(c) the driver cannot travel both ways by the same route?

4.14)



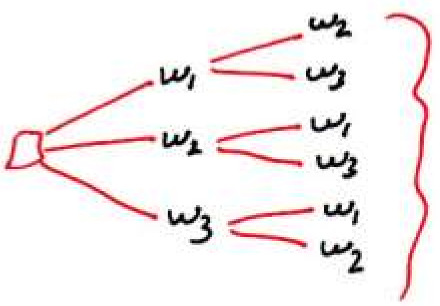
a) Same Route → $\{(w_1,w_1), (w_2,w_2), (w_3,w_3)\}$ OR,



3 different ways.

b) $n(U) = 3$; $n(D) = 3$; in $\underline{3} \cdot \underline{3} = 9$ different ways

c)



6 different ways.

OR; $\underline{3} \cdot \underline{2} = 6$

↳ do NOT use the same way, Remaining = 2

4.9 A shopper wants to visit three of five department stores and wants to decide which stores she will visit first, second, and third. She will not visit the same store twice. In how many ways can she select the three stores?

4.9) $S = \{s_1, s_2, s_3, s_4, s_5\}$ $n(S) = 5$

$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$ different ways.

4.15 A bank identifies its charge cards by serial numbers consisting of two letters of the alphabet followed by five digits. How many identifiable charge cards can the bank issue under this plan?

4.15) (_ _ / _ _ _ _ _) Letters → 26
Digits → 10

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^5 = 67600000$

identifiable charge cards.

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4.5 A quiz consists of 6 multiple choice questions, each with five choices.

(a) In how many ways can a student mark the answers to the questions if one choice is made for each of the questions?

(b) In how many ways can a student get a perfect score on the quiz?

(c) In how many ways can a student mark the answers to all the questions, marking all of them incorrectly?

4.5) a) $\underline{5}\cdot\underline{5}\cdot\underline{5}\cdot\underline{5}\cdot\underline{5}\cdot\underline{5} = 5^6 = 15625$

b) $\underline{1}\cdot\underline{1}\cdot\underline{1}\cdot\underline{1}\cdot\underline{1}\cdot\underline{1} = 1^6 = 1$

c) $\underline{4}\cdot\underline{4}\cdot\underline{4}\cdot\underline{4}\cdot\underline{4}\cdot\underline{4} = 4^6 = 4096$

Permutations

* We can order a set of n elements in $n!$ ways.

Example; In how many ways 5 students line up?

Answer; $\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 5! = 120$

4.29 (a) In how many ways can a librarian arrange six different books on a shelf?

(b) In how many ways can six truck drivers be assigned to six company trucks?

(c) In how many ways can six architectural designs for a building be ranked in order of preference?

a) $6!$ b) $6!$ c) $6!$

* We can (select and) order r elements from a set of n elements in ${}_nP_r = \frac{n!}{(n-r)!} = n\cdot(n-1)\cdot\ldots\cdot(n-r+1)$ different ways.

Note that the idea here is, since we ORDER elements, a change between two of the elements will yield a different permutation!

4.26 In how many ways can a fleet commander assign five ships to the first, second, third, fourth and fifth positions in a column of ten ships?

4.26) ${}_{10}P_5 = \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} = 30240$

4.33 Find the number of permutations of the letters of the following words:
(a) lullaby;
(b) loophole;
(c) paperback;
(d) Mississippi.

4.33) a) If all letters were different, we would have 7! different words by changing the letters of lullaby (Consider l_1 u l_2 l_3 a b y)

However, by changing the places of l's, we won't have obtained a different word. l's can change places within each other in 3! different ways.

So, the answer is $\frac{7!}{3!} = 840$

b) loophole → 8 letters
l → 2 ; o → 3 ; other letters → 1
The answer is, $\frac{8!}{2! \cdot 3!} = 3360$

c) Mississippi → 11 letters
i → 4 ; s → 4 ; p → 2 ⇒ $\frac{11!}{4! \cdot 4! \cdot 2!} = 34650$

Combinations

We can select r elements from a set of n elements in ${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{{}_nP_r}{r!}$

Here, order of the selection is NOT important.

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4.39 Symbolically, we let $\binom{n}{r}$ denote the number of combinations of r objects selected from a set of n objects. Express the following situations using the $\binom{n}{r}$ symbol. Do not do the arithmetic.

(a) A social club with 25 members wants to choose 5 members for the entertainment committee.

(b) An Army sergeant wants to pick 4 privates in a 14-man platoon for a work detail.

(c) The captain of a police detective unit wants to select 3 detectives from among his 10 detectives to form a special investigation team.

(d) A scientist wants to pick 6 of 18 guinea pigs from a cage to be subjects of an experiment.

4.39) a) $\binom{25}{5} = \frac{25.24.23.22.21}{5!} = 53130$ b) $\binom{14}{4} = \frac{14.13.12.11}{4!} = 1001$

c) $\binom{10}{3} = \frac{10.9.8}{3!} = 120$ d) $\binom{18}{6} = \frac{18.17.16.15.14.13}{6!} = 18564$

4.40 A stock brokerage firm plans to reduce the size of its work force by dismissing two of its nine brokers. If five brokers are men and the rest are women, in how many ways can the manager dismiss

(a) any two of the male brokers;

(b) any two of the female brokers;

(c) one male and one female broker;

(d) any two brokers?

4.40) $\left.\begin{matrix}\text{5 men}\\ \text{4 Women}\end{matrix}\right\}$ 9 workers

d) $\binom{9}{2} = \frac{9.8}{2!} = 36$

a) $\binom{5}{2} = \frac{5.4}{2!} = 10$ b) $\binom{4}{2} = \frac{4.3}{2!} = 6$ c) $\binom{5}{1}.\binom{4}{1} = 5.4 = 20$

4.41 A carton of 12 transistor batteries contains 4 that are defective. In how many different ways can one choose 3 of these batteries so that

(a) none of the defective batteries is included;

(b) exactly 1 of the defective batteries is included;

(c) exactly 2 of the defective batteries are included;

(d) exactly 3 of the defective batteries are included?

4 def
8 NON def
12 batteries
select
n=3

a) $\binom{8}{3} = 56$ b) $\binom{4}{1}\binom{8}{2} = 112$ c) $\binom{4}{2}\binom{8}{1} = 48$ d) $\binom{4}{3} = 4$

4.42 A housing construction firm plans to fill three carpenter's positions from a pool of five applicants; two plumbers positions from a group of four applicants; and four electricians from six applicants. If the order does not matter, in how many ways can a foreman hire these nine craftspeople? Do not use Table X.

4.42) Carpenter: 3 out of 5
Plumder: 2 out of 4
Electriciens: 4 out of 6

$\Rightarrow$ Overall Selection;

$$\binom{5}{3}\cdot\binom{4}{2}\cdot\binom{6}{4}=10\cdot 6\cdot 15=900$$

Probability

S: Sample space is the set that contains all possible outcomes of the experiment conducted.

Let A be a subset of S.

The probability of Event A is defined as;

$$P(A)=\frac{\text{Number of times A occurs}}{\text{Number of all possible outcomes}}=\frac{n(A)}{n(S)}$$

4.51 If 1 card is drawn from a well-shuffled deck of 52 playing cards, what are the probabilities of getting

(a) a red king;

(b) a black card;

(c) a 3, 4, 5, or 6;

(d) a diamond;

(e) not a diamond;

(f) not an ace?

4.51) Playing Cards;

Numbers: A 2 3 4 5 6 7 8 9 10

FACES: J Q K

SUITS:
- BLACK: Spade ♤, Club ♧
- RED: Heart ♡, Diamond ♢

4.51) a) $P(\text{Red King}) = \frac{2}{52}$ b) $P(\text{Black}) = \frac{26}{52}$

c) $P(3, 4, 5 \text{ or } 6) = \frac{4 \cdot 4}{52} = \frac{16}{52}$ d) $P(\text{Diamond}) = \frac{13}{52}$

e) $P(\text{Not a Diamond}) = 1 - P(\text{Diamond}) = 1 - \frac{13}{52} = \frac{39}{52}$

f) $P(\text{Not an Ace}) = 1 - P(\text{Ace}) = 1 - \frac{4}{52} = \frac{48}{52}$

4.53 If H stands for heads and T for tails, the eight possible outcomes for three flips of a coin are HHH, HHT, HTH, THH, HTT, THT, TTH, and TTT. If it can be assumed that these eight possibilities are equally likely, what are the probabilities of getting 0, 1, 2, or 3 heads?

4.53) Let X is Number of heads obtained. This is called a Random Variable, we'll see it later in detail.

All outcomes are equally likely $= \frac{1}{8}$

We have;

$P(X=0) = P\{TTT\} = \frac{1}{8}$; $P(X=1) = P\{HTT, THT, TTH\} = \frac{3}{8}$

$P(X=2) = P\{HHT, HTH, THH\} = \frac{3}{8}$; $P(X=3) = P\{HHH\} = \frac{1}{8}$

The probability distribution of a Random Variable X is the function which assigns probabilities to the values of the Random Variable X

X	0	1	2	3
p(x)	1/8	3/8	3/8	1/8

4.56 If 2 cards are drawn from a well-shuffled deck of 52 playing cards, what are the probabilities of getting

(a) two spades;

(b) two aces;

(c) a king and a queen?

4.56) a) $P(\text{Two Spades}) = \dfrac{\binom{13}{2}}{\binom{52}{2}} = \dfrac{78}{1326} = 0{,}0588$

b) $P(\text{Two Aces}) = \dfrac{\binom{4}{2}}{\binom{52}{2}} = \dfrac{6}{1326} = 0{,}0045$

c) $P(\text{A King and A Queen}) = \dfrac{\binom{4}{1}\binom{4}{1}}{\binom{52}{2}} = \dfrac{16}{1326} = 0{,}0121$

4.59 A builder has 20 slabs of gray granite and 10 slabs of pink granite for use in the construction of a building. If 3 slabs of granite are randomly selected, what are the probabilities that

(a) all will be pink;

(b) 2 will be gray and 1 will be pink?

4.59) $\boxed{\begin{array}{l}20 \text{ gray}\\ 10 \text{ pink}\end{array}} \xrightarrow{\text{select}} n=3$

30 granites

a) $P(3 \text{ Pinks}) = \dfrac{\binom{10}{3}}{\binom{30}{3}} = \dfrac{120}{4060} = 0{,}0296$

b) $P(2 \text{ Gray}, 1 \text{ Pink}) = \dfrac{\binom{20}{2}\binom{10}{1}}{\binom{30}{3}} = \dfrac{1900}{4060} = 0{,}4680$

* Also note that, if we let X: # of Pinks obtained, we have,

$P(1 \text{ Gray}, 2 \text{ Pinks}) = \dfrac{\binom{20}{1}\binom{10}{2}}{\binom{30}{3}} = \dfrac{900}{4060} = 0{,}2217$

$P(3 \text{ Gray}, 0 \text{ Pink}) = \dfrac{\binom{20}{3}}{\binom{30}{3}} = \dfrac{1140}{4060} = 0{,}2807$

Then, the probability distribution of X is;

X	0	1	2	3
$p(x)$	0,2807	0,4680	0,2217	0,0296

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4.64 According to the American Medical Association, there were 797,634 physicians in the United States in the year 1999 of whom 611,028 were male and 186,606 were female. Of these physicians, 27,790 were male psychiatrists and 11,266 were female psychiatrists.

(a) What is the probability that a randomly selected physician is male?

(b) What is the probability that a randomly selected physician is female?

(c) What is the probability that a randomly selected physician is a psychiatrist?

(d) What is the probability that a randomly selected physician is not a psychiatrist?

(e) What is the probability that a randomly selected psychiatrist is male?

4.64)

	Male	Female	TOTAL
Psychiatrist	27790	11266	27790+11266 = 39056
NOT Psychiatrist	611028−27790 = 583238	186606−11266 = 175340	797634−39056 = 758578
TOTAL	611028	186606	797634

a) $P(\text{Male}) = \frac{611028}{797634} = 0{,}7661$ b) $P(\text{Female}) = \frac{186606}{797634} = 0{,}2339$

c) $P(\text{Psychiatrist}) = \frac{39056}{797634} = 0{,}04896$

d) $P(\text{Not Psychiatrist}) = 1 - P(\text{Psychiatrist})$

$= 1 - 0{,}04896 = 0{,}95104$

e) $P(\text{Male} \mid \text{Psychiatrist}) = \frac{11266}{39056} = 0{,}2885$

Some Rules of Probability

* Sample space is the set of all possible outcomes

A subset of sample space is called an event

$\boxed{A \subset S}$

$P(A)$: Probability of Event A

Impossible Event $\leftarrow$ $0 \leq P(A) \leq 1$ $\rightarrow$ Certain Event

* What is probability?

Probability of an event is its "likelihood." Formally, probability of an event is its long-run fraction of occurence.

Remember;

$$P(A) = \frac{\text{Number of times event } A \text{ occurs.}}{\text{Number of all possible outcomes.}}$$

If elements of sample space are "equally likely";

$$P(A) = \frac{n(A)}{n(S)}$$

5.5 If one card is drawn from an ordinary deck of 52 playing cards, the sample space may be written as

$$S = \begin{matrix} \{A\spadesuit, & 2\spadesuit, & 3\spadesuit, & \ldots, & K\spadesuit, \\ A\heartsuit, & 2\heartsuit, & 3\heartsuit, & \ldots, & K\heartsuit, \\ A\diamondsuit, & 2\diamondsuit, & 3\diamondsuit, & \ldots, & K\diamondsuit, \\ A\clubsuit, & 2\clubsuit, & 3\clubsuit, & \ldots, & K\clubsuit\} \end{matrix}$$

where $\spadesuit$, $\heartsuit$, $\diamondsuit$, and $\clubsuit$ denote the suits spades, hearts, diamonds, and clubs. If

$$M = \{Q\spadesuit, \ K\spadesuit, \ Q\heartsuit, \ K\heartsuit, \ Q\diamondsuit, \ K\diamondsuit, \ Q\clubsuit, \ K\clubsuit\}$$

and

$$N = \{10\spadesuit, \ J\spadesuit, \ Q\spadesuit, \ K\spadesuit\}$$

We have, M: {Queen or King}

N: {Spade 10 or Spade Face}

$$P(M) = \frac{n(M)}{n(S)} = \frac{8}{52} \quad \text{and} \quad P(N) = \frac{n(N)}{n(S)} = \frac{4}{52}$$

Union, Intersection and Complement of Events

Union: $\cup$ means "OR": At least one of the events

Intersection: $\cap$ means "AND": Two events together

Complement: means "NOT": A': NOT A

From set theory, we have,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and

$P(A') = 1 - P(A)$. $(P(\bar{A}) = 1 - P(A))$

Also NOTE that, $(A \cap B)' = A' \cup B'$ and

$(A \cup B)' = A' \cap B'$

So, $P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B)$

Return to Exercise 5.5)

M and N $M \cap N = \{♠Q ; ♠K\}$, $P(M \cap N) = \frac{2}{52}$.

M or N $P(M \cup N) = P(M) + P(N) - P(M \cap N) = \frac{8}{52} + \frac{4}{52} - \frac{2}{52} = \frac{10}{52}$

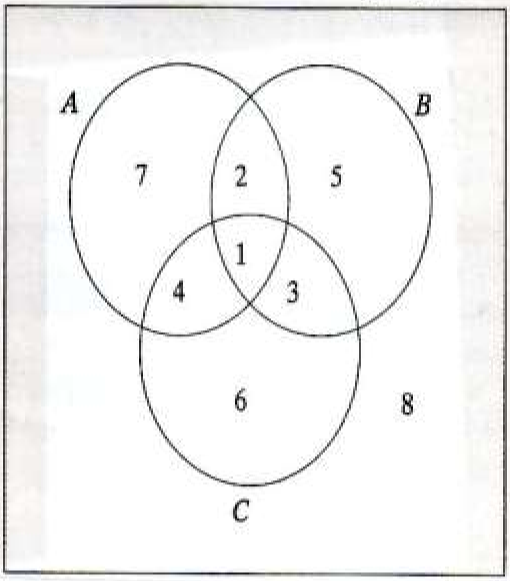
NOT M $P(M') = 1 - P(M) = 1 - \frac{8}{52} = \frac{44}{52}$; $P(N') = 1 - \frac{4}{52} = \frac{48}{52}$

Neither M nor N $P(M' \cap N') = P[(M \cup N)'] = 1 - P(M \cup N) = 1 - \frac{10}{52} = \frac{48}{52}$

Venn Diagram

5.17 Continuing with Exercise 5.16, explain in words what the following regions represent:

(a) region 1;
(b) region 3;
(c) region 6;
(d) region 8;
(e) regions 1 and 4 together;
(f) regions 3 and 5 together;
(g) regions 1, 3, 4, and 6, together;
(h) regions 2, 5, 7, and 8 together.



5.17)
a) $A \cap B \cap C$
b) $(B \cap C) \cap A'$
c) $C \cap (A \cup B)'$
d) $(A \cup B \cup C)'$
e) $A \cap C$
f) $B \cap A'$
g) C
h) C'

5.21 The probabilities that a computer store will sell 0, 1, 2, 3, or at least 4 computers on a typical business day are 0.10, 0.15, 0.20, 0.25, and 0.30. What are the probabilities that, on a typical day.

(a) at most three computers will be sold;
(b) at least two computers will be sold;
(c) two or three computers will be sold?

5.21) Let X: # of computers sold in a day. Then,

x	0	1	2	3	4 or more
$p(x)$	0,10	0,15	0,20	0,25	0,30

a) $P(X \leq 3) = 1 - P(X = 4) = 1 - 0{,}30 = 0{,}70$

b) $P(X \geq 2) = 0{,}20 + 0{,}25 + 0{,}30 = 0{,}75$

c) $P(X = 2 \text{ or } X = 3) = P(X=2) + P(X=3) = 0{,}20 + 0{,}25 = 0{,}45$

5.27 The probability that a hotel manager's reaction to a new musical tape played continuously in the lobby is 0.55 that the volume of sound is too high, and 0.10 that the volume is just right. Find the probability that the manager's reaction to the volume is that it is

(a) too low;
(b) too low or just right;
(c) too high or just right.

5.27) P (Too High) = 0,55 ; P(Just Right) = 0,10

a) P(Too Low) = 1 − 0,55 − 0,10 = 0,35

b) P(Too Low OR Just Right) = 0,35 + 0,10 = 0,45

c) P(Too High OR Just Right) = 0,55 + 0,10 = 0,65

* If two events cannot happen together, they are called mutually exclusive events. Namely, if A and B are mutually exclusive, then $P(A \cap B) = 0$

5.24 Given the *mutually exclusive* events Y and Z, for which $P(Y) = 0.28$ and $P(Z) = 0.47$, find

(a) $P(Y')$; d) $P(Y \cup Z)$;
(b) $P(Z')$; (e) $P(Y' \cap Z')$.
(c) $P(Y \cap Z)$;

7.24) $P(Y) = 0{,}28$; $P(Z) = 0{,}47$

a) $P(\bar{Y}) = 1 - P(Y) = 1 - 0{,}28 = 0{,}72$

b) $P(\bar{Z}) = 1 - P(Z) = 1 - 0{,}47 = 0{,}53$ c) $P(Y \cap Z) = 0$

d) $P(Y \cup Z) = P(Y) + P(Z) - P(Y \cap Z) = 0{,}28 + 0{,}47 - 0 = 0{,}75$

e) $P(\bar{Y} \cap \bar{Z}) = P(\overline{Y \cup Z}) = 1 - P(Y \cup Z) = 1 - 0{,}75 = 0{,}25$

* The ODDS that an event will occur are given by the ratio of the probability that will occur to the probability that it will NOT occur.

Namely, "the odds for occurance of an event is a to b" means; $\boxed{\frac{a}{b} = \frac{p}{1-p}}$

where p is the probability of its occurance.

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5.32 Convert each of the following odds to probabilities:

(a) The odds that a particular horse will lose a race are 7 to 1.

(b) The odds are 3 to 5 that a sequence of four coin tosses will result in two heads and two tails.

(c) If a secretary randomly places six letters into six addressed envelopes, the odds are 1 to 719 that all letters will end up in the correct envelopes.

(d) The odds are 2 to 17 for winning a roulette bet made by placing a token at the intersection of four number boxes.

5.33 Convert each of the following probabilities to odds:

(a) The probability that the last digit of a postal zip code is 5, 6, 7, 8, 9, or 0 is $\frac{6}{10}$.

(b) The probability of randomly selecting the 8 letters alpha, beta, gamma, delta, epsilon, zeta, eta, theta from the 24 letters of the Greek alphabet is $\frac{8}{24}$.

(c) The probability of getting 2 heads in 4 flips of a coin is $\frac{6}{16}$.

(d) The probability of drawing a heart from a randomly shuffled deck of 52 playing cards is $\frac{13}{52}$.

5.32) a) Let p: Horse will win the race

$$\frac{7}{1} = \frac{1-p}{p} \Rightarrow 7p = 1-p \Rightarrow 8p = 1 \Rightarrow p = \frac{1}{8}$$

b) P(2 Heads and Two tails) $= p$

$$\frac{3}{5} = \frac{p}{1-p} \Rightarrow 3-3p = 5p \Rightarrow 8p = 3 \Rightarrow p = \frac{3}{8}$$

c) p: Probability to end up with correct envelopes

$$\frac{1}{719} = \frac{p}{1-p} \Rightarrow p = \frac{1}{720}$$

d) p: Probability of winning a roulette bed, $\frac{p}{1-p} = \frac{2}{17} \Rightarrow p = \frac{2}{19}$

5.33) a) $\frac{a}{b} = \frac{p}{1-p} = \frac{6/10}{4/10} = \frac{6}{4}$: 6 to 4 (or 3 to 2) b) $\frac{a}{b} = \frac{8/24}{16/24} = \frac{8}{16}$: 1 to 2

c) $\frac{a}{b} = \frac{6/16}{10/16} = \frac{6}{10} = \frac{3}{5}$: 3 to 5 d) $\frac{a}{b} = \frac{13/52}{39/52} = \frac{13}{39} = \frac{1}{3}$: 1 to 3

5.42 A professor of English with very large classes anticipates that the percentages of students who will receive grades of A, B, C, D, or F are, respectively, 6, 22, 44, 22, and 6.

(a) What is the probability that a student will get a grade of C or higher?

(b) What is the probability that a student will get a grade of C or lower?

(c) What is the probability that a student will get a grade lower than A but higher than F?

5.42)

Grade	A	B	C	D	F
P(Grade)	0,06	0,22	0,44	0,22	0,06

a) P(C OR Higher) = 0,44 + 0,22 + 0,06 = 0,72

b) P(C OR Lower) = 0,44 + 0,22 + 0,06 = 0,72

c) P(Lower than A but Higher than F) = P(B, C OR D)

= 0,22 + 0,44 + 0,22 = 0,88

5.53 Among the 64 doctors on the staff of a hospital, 58 carry malpractice insurance, 33 are surgeons, and 31 of the surgeons carry malpractice insurance. If one of these doctors is chosen by lot to represent the hospital staff at an American Medical Association (AMA) convention (that is, each doctor has a probability of $\frac{1}{64}$ of being selected), what is the probability that the one chosen is not a surgeon and does not carry malpractice insurance?

5.54 Given $P(A) = 0.59$, $P(B) = 0.46$, and $P(A \cap B) = 0.28$, draw a Venn diagram, fill in the probabilities associated with the various regions, and thus determine

(a) $P(A' \cap B)$; (c) $P(A \cup B)$;
(b) $P(A \cap B')$; (d) $P(A' \cap B')$.

5.53)

	Carry	NOT Carry	TOTAL
Surgeon	31	33-31=2	33
NOT Surgeon	58-31=27	6-2=4	27+4=31
TOTAL	58	64-58=6	64

P(NOT Surgeon AND NOT carry) = $\frac{4}{64} = \frac{1}{16}$

5.54)

$P(A' \cap B) = 0,16$ $P(A \cup B) = 0,75$

$P(A \cap B') = 0,31$ $P(A' \cap B') = 0,25$

5.50 The probabilities that a homeowner will repair the roof, paint the house, or both are, respectively, 0.90, 0.58, and 0.50. What is the probability that the homeowner will fix the roof or paint the house?

5.51 The probabilities that a jeweler will sell a large diamond ring, a large sapphire ring, or both on a given day are, respectively, 0.06, 0.08, and 0.03. What is the probability that at least one of the rings will be sold by the jeweler on that day?

5.50) R: Repair the Roof, H: Paint the House

$P(R) = 0{,}90$; $P(H) = 0{,}58$; $P(R \cap H) = 0{,}50$

$P(R \text{ or } H) = P(R \cup H) = P(R) + P(H) - P(R \cap H)$

$= 0{,}90 + 0{,}58 - 0{,}50 = 0{,}98$

5.51) D: Large Diamond Ring; S: Large sapphire ring

$P(D) = 0{,}06$; $P(S) = 0{,}08$; $P(D \cap S) = 0{,}03$

$P(\text{At least one}) = P(D \cup S) = P(D) + P(S) - P(D \cap S)$

$= 0{,}06 + 0{,}08 - 0{,}03 = 0{,}11$

Conditional Probability & Independence:

* Basically, there are 3 types of probability;

(i) Marginal → Single Event

(ii) Joint → Two Events Together

(iii) Conditional → An event under occurance of another event

Ex: A study on color choices and gender is shown in the following table:

Gender \ Color choice	Pink	Blue	White	TOTAL
Male	6	16	8	40
Female	35	15	10	60
TOTAL	41	31	18	100

What is the probability that (WPT);

a) A randomly choosen person is male?

b) A randomly choosen person selects White?

c) A randomly choosen person is Female and selects Blue?

d) If it is known that color choice is Pink, WPT the person is female?

e) WPT a female chooses White?

Ans// (i) Marginal

a) $P(M) = \frac{40}{100}$ b) $P(W) = \frac{18}{100}$

(ii) Joint

c) $P(F \cap B) = \frac{15}{100}$

(iii) Conditional

d) $P(F|P) = \frac{35}{41}$ e) $P(W|F) = \frac{10}{60}$

* Consider the probability in part (e)

$$P(W|F) = \frac{10}{60} = \frac{10/100}{60/100} = \frac{P(W \cap F)}{P(F)}$$

In General; $\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$

given

$P(A|B)$: Probability of A given B

Known Event

Random Event in (the knowledge of) B

* By Cross Multiplication, we have:

$$\boxed{P(A \cap B) = P(B) \cdot P(A|B)}$$ ↳ Multiplication Rule for Any events A and B

5.65 In the following table, 60 college students are classified according to their class standing and also according to their favorite pizza topping:

	(A) Anchovies	(O) Onions	(M) Mushrooms	(H) Hamburger	TOTAL
(F) Freshman	7	6	7	3	23
(S) Sophomore	1	9	0	9	19
(J) Junior	3	2	5	8	18
TOTAL	11	17	12	20	60

If one of these students is selected at random, if F, S, and J denote the three classes, and if A, O, M, and H denote the four pizza toppings, find

(a) $P(M \cup J)$; (d) $P(F'|A)$;
(b) $P(H|F)$; (e) $P(M \cup H|J')$;
(c) $P(O \cap S)$; (f) $P(J|A \cup M)$.

5.66 With reference to Exercise 5.65, find the probabilities that the student chosen will be

(a) a freshman whose favorite pizza topping is mushrooms;

(b) an anchovy pizza eater given that he or she is a junior;

(c) a sophomore given that he or she is not a junior.

5.65) a) $P(M \cup J) = P(M) + P(J) - P(M \cap J) = \frac{12}{60} + \frac{18}{60} - \frac{5}{60} = \frac{25}{60}$

b) $P(H|F) = \frac{3}{23}$ c) $P(O \cap S) = \frac{9}{60}$

d) $P(\bar{F}|A) = 1 - P(F|A) = 1 - \frac{7}{11} = \frac{4}{11}$

e) $P(M \cup H|\bar{J}) = \frac{P[(M \cup H) \cap \bar{J}]}{P(\bar{J})} = \frac{7+3+0+9}{23+19} = \frac{19}{42}$

f) $P(J|A \cup M) = \frac{3+5}{11+12} = \frac{8}{23}$

5.60 The probability that a security guard will be hired at a shopping mall is 0.80, and the probability that the security guard will be hired and will decrease the number of thefts is 0.75. What is the probability that, if the security guard is hired, the number of thefts will decrease?

5.60) $P(\text{Hired}) = 0{,}80$; $P(\text{Hired} \cap \text{Decrease Thefts}) = 0{,}75$

$$P(\text{Decrease Thefts} \mid \text{Hired}) = \frac{0{,}75}{0{,}80} = \underline{\underline{0{,}9375}}$$

* Two events A and B are independent if knowledge of occurance of one event does NOT change the probability of the other event. In other words, A and B are independent if

$$\boxed{P(A|B) = P(A)} \quad (\text{OR } P(B|A) = P(B))$$

Then, for independent events A and B; we have,

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$ Multiplication Rule for independent Events.

5.59 If $P(A) = 0.70$, $P(B) = 0.40$, and $P(A \cap B) = 0.25$, are events A and B independent?

5.59) $P(A) \cdot P(B) = 0{,}70 \cdot 0{,}40 = 0{,}28 \neq 0{,}25 = P(A \cap B)$

So, A and B are NOT independent.

5.57 If X and Y are independent events and $P(X) = 0.25$ and $P(Y) = 0.50$, find

(a) $P(X|Y)$;
(b) $P(X \cap Y)$;
(c) $P(X \cup Y)$;
(d) $P(X' \cap Y')$.

5.57) $P(X)=0{,}25$; $P(Y)=0{,}50$; X and Y are independent

a) $P(X/Y)=P(X)=0{,}25$

b) $P(X\cap Y)=P(X).P(Y)=0{,}25.0{,}50=0{,}125$

c) $P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)=0{,}25+0{,}50-0{,}125=0{,}625$

d) $P(\bar{X}\cap\bar{Y})=P(\overline{X\cup Y})=1-P(X\cup Y)=1-0{,}625=0{,}375$

5.69 Assume that the following are all independent events, and calculate their probabilities.

(a) The probability of getting five heads in a row with a balanced coin.

(b) The probability of drawing three clubs in a row (with replacement) from an ordinary deck of 52 playing cards.

(c) The probability of drawing three clubs in a row (without replacement) from an ordinary deck of playing cards.

(d) The probability that a shooter firing at a target will, in two consecutive shots, hit the target once and then miss the target. Assume that the probability that the shooter hits the target on any one try is 0.80.

5.69) a) $P(HHHHH)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\left(\frac{1}{2}\right)^5=\frac{1}{32}$

b) $P(CCC)=\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}=\left(\frac{1}{4}\right)^3=\frac{1}{64}$

c) $P(C_1C_2C_3)=P(C_1).P(C_2|C_1).P(C_3|(C_1\cap C_2))$

$$=\frac{13}{\cancel{52}_{13}}\cdot\frac{\cancel{12}^{3}}{51}\cdot\frac{\cancel{10}^{1}}{\cancel{50}_{5}}=\frac{3}{255}=\frac{1}{85}$$

d) $P(Hit\cap Miss)=0{,}80.(1-0{,}80)=0{,}16$

5.76 What is the probability of *not* rolling a six

(a) in a single roll of a balanced die;

(b) in two rolls of a balanced die;

(c) in three rolls of a balanced die?

5.76) $P(\text{Not Rolling a six})=1-\frac{1}{6}=\frac{5}{6}$

a) $\frac{5}{6}$ b) $\frac{5}{6}\cdot\frac{5}{6}=\frac{25}{36}$

c) $\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=\frac{125}{216}$

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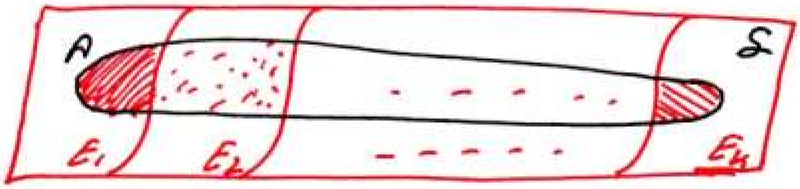
Total Probability Rule & BAYES' Theorem

* Let $E_1, E_2, \ldots, E_k$ are mutually exclusive & collectively exhaustive (means $\bigcup_{i=1}^{k} E_i = S$) events

Also let A be another event in S:



$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \text{----} + P(A \cap E_k)$$

$$\boxed{P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + \text{----} + P(E_k).P(A|E_k)}$$

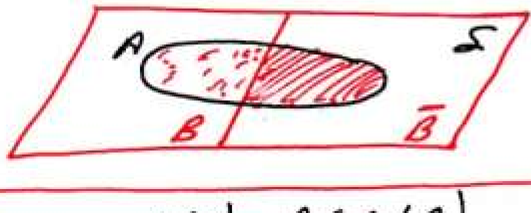
TOTAL Probability Rule

Then; $$P(E_j|A) = \frac{P(E_j \cap A)}{P(A)}$$

$$\boxed{P(E_j|A) = \frac{P(E_j).P(A|E_j)}{P(E_1).P(A|E_1) + \text{----} + P(E_k).P(A|E_k)}}$$

BAYES Theorem.

* Also Remember, B and $\bar{B}$ are like E_1 and E_2. So;



$$\boxed{P(B|A) = \frac{P(B).P(A|B)}{P(B).P(A|B) + P(\bar{B}).P(A|\bar{B})}}$$

5.87 In the billing department of a local city, Arthur, Beatrice, and Carla prepare and mail out 50%, 30%, and 20% of the real-estate tax bills, respectively, to property owners. The bills are verified by the treasurer before mailing. If 0.5% of the tax bills prepared by Arthur, 0.3% of the tax bills prepared by Beatrice, and 1.0% of the tax bills prepared by Carla are inaccurate, what are the probabilities that an inaccurate statement detected by the treasurer at final verification was prepared by

(a) Arthur;

(b) Beatrice;

(c) Carla?

5.87)

$P(A) = 0{,}50 \;;\; P(D|A) = 0{,}005$

$P(B) = 0{,}30 \;;\; P(D|B) = 0{,}003$

$P(C) = 0{,}20 \;;\; P(D|C) = 0{,}01$

A: Arthur B: Beatrice C: Carla D: inaccurate bills

$$P(D) = 0{,}50 \cdot 0{,}005 + 0{,}30 \cdot 0{,}003 + 0{,}20 \cdot 0{,}01 = 0{,}0054$$

a) $$P(A|D) = \frac{0{,}50 \cdot 0{,}005}{0{,}0054} = 0{,}463$$

b) $$P(B|D) = \frac{0{,}30 \cdot 0{,}003}{0{,}0054} = 0{,}167$$

c) $$P(C|D) = \frac{0{,}20 \cdot 0{,}01}{0{,}0054} = 0{,}370$$

5.89 A driver's license examiner knows that 75% of all applicants have attended a driving school. If an applicant has attended a driving school, the probability is 0.85 that he or she will pass the license examination, and if an applicant has not attended a driving school, the probability is 0.60 that he or she will pass the examination. If an applicant passes the license examination, what is the probability that he or she has attended a driving school?

5.89) A: Attend a driving school, E: Pass the licence Exam

$P(A) = 0{,}75 \;;\; P(E/A) = 0{,}85$

$P(\bar{A}) = 0{,}25 \;;\; P(E/\bar{A}) = 0{,}60$

$$P(E/A) = \frac{0{,}75 \cdot 0{,}85}{0{,}75 \cdot 0{,}85 + 0{,}25 \cdot 0{,}60} = 0{,}8095$$