STATISTICS for Lawyers & Social Sciences | CHAPTERS 6&7

Discrete Probability Distributions

Remember, X is called a "Random Variable" if it defines a function from Sample Space of an experiment to Real Numbers.

For example, if we flip a coin 4 times and we are interested in "Number of Heads obtained", letting X: # of Heads obtained, we have

$$X(HTTT) = X(THTT) = X(TTHT) = X(TTTH) = 1$$

Note that, we are NOT interested in the actual outcome. What interests us is How Many of the flips resulted in "Heads". In the example above, 4 different outcomes cause $X = 1$.

If X takes only Numbers, X is a discrete Random Variable.

$f(x)$: Probability mass function (pmf) of a discrete random Variable assigns probabilities to the Values of X can take. $f(x)$ is valid only if;

(i) $f(x) \geq 0$ → Nonnegative Probabilities

(ii) $\sum_x f(x) = 1$ → Total Probability is 1.

We have, $P(\underbrace{X}_{\text{Random VARIABLE}} = \underbrace{x}_{\text{Numerical Value}}) = f(x)$

Alp Giray Özen | 0533 549 91 08 | alp@lecturemania.com | www.lecturemania.com

6.2 Check in each case whether the given function can serve as the probability distribution of an appropriate random variable.

(a) $f(x) = \frac{1}{5}$ for $x = 0, 1, 2, 3, 4, 5$;

(b) $f(x) = \frac{x^2 - 1}{25}$ for 0, 1, 2, 3;

(c) $f(x) = \frac{x^2}{14}$ for $x = 0, 1, 2, 3$.

6.3 In each case determine whether the given values can be looked upon as the values of a probability distribution of a random variable that can take on the values 1, 2, 3, and 4. Explain your answers.

(a) $f(1) = 0.25, f(2) = 0.30, f(3) = 0.35, f(4)\ 0.40$;

(b) $f(1) = 0.05, f(2) = 0.10, f(3) = 0.15, f(4) = 0.20$;

(c) $f(1) = 0.10, f(2) = 0.20, f(3) = 0.30, f(4) = 0.40$;

(d) $f(1) = -0.10, f(2) = -0.20, f(3) = -0.30; f(4) = -0.40$.

6.2) a) (i) $f(x) = \frac{1}{5} \geq 0$ for $x = 0, 1, \dots, 5$ ✓

(ii) $\sum_{x=0}^{5} f(x) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{6}{5} \neq 1$ ✗

$f(x)$ is NOT a pmf

b) (i) $f(0) = \frac{0^2 - 1}{25} = \frac{-1}{25} < 0$ ✗

$f(x)$ is NOT a pmf

c) (i) $f(x) = \frac{x^2}{14} \geq 0$ for $x = 0, 1, 2, 3$ ✓

(ii) $\sum_{x=0}^{3} f(x) = \sum_{x=0}^{3} \frac{x^2}{14} = \frac{0}{14} + \frac{1}{14} + \frac{4}{14} + \frac{9}{14} = \frac{14}{14} = 1$ ✓

$f(x)$ is a pmf

6.3) a) $\sum f(x) = 1{,}30 \neq 1$ No

b) $\sum f(x) = 0{,}50 \neq 1$ No

c) (i) $f(x) \geq 0$ ✓

(ii) $\sum f(x) = 0{,}1 + 0{,}2 + 0{,}3 + 0{,}4 = 1$ ✓ YES

d) $f(1) = -0{,}10 < 0$ No

Mean and Standard Deviation.

Mean of a discrete Random Variable X is its long run average Value. Note that, "Mean" "Average" and "Expected Value" have the same meaning: μ

$$\boxed{\mu = E(X) = \sum x \cdot f(x)}$$ → Sumproduct of Values X can take and their respective probabilities.

Variance is square of standard deviation. Both variance and standard deviation are "dispertion parameters" which shows a measure "how are the values of X away from their means"

σ^2 : Variance ; σ = Standard deviation

We have, $\mu = E(X) = \sum x \cdot f(x)$

$$E(X^2) = \sum x^2 \cdot f(x)$$

$$\boxed{\sigma^2 = Var(X) = E(X^2) - \mu^2}$$

$$\sigma = \sqrt{\sigma^2}$$

6.62 Suppose that the probabilities are 0.50, 0.25, 0.15 and 0.10 that 0, 1, 2, or 3 students are absent from a certain class on any day.

(a) Find the mean of this probability distribution.

(b) Find the variance of this probability distribution.

6.63 The following table gives the probabilities that a jeweler will sell 0, 1, 2, 3, 4, or 5 wrist watches on any given day.

Number of watches	0	1	2	3	4	5
Probability	0.05	0.20	0.30	0.25	0.15	0.05

(a) Find the mean of this probability distribution.

(b) Find the standard deviation of this probability distribution.

6.62) X: # of absent student on a day

X	0	1	2	3
f(x)	0,50	0,25	0,15	0,10

a) $\mu = E(X) = \sum x \cdot f(x) = 0 \cdot 0{,}50 + 1 \cdot 0{,}25 + 2 \cdot 0{,}15 + 3 \cdot 0{,}10$
$= \underline{\underline{0{,}85}}$

b) $E(X^2) = \sum x^2 \cdot f(x) = 0^2 \cdot 0{,}50 + 1^2 \cdot 0{,}25 + 2^2 \cdot 0{,}15 + 3^2 \cdot 0{,}10 = 1{,}75$

$\sigma^2 = Var(X) = E(X^2) - \mu^2 = 1{,}75 - 0{,}85^2 = \underline{\underline{1{,}0275}}$

6.63) X: Number of watches

X	0	1	2	3	4	5
f(x)	0,05	0,20	0,30	0,25	0,15	0,05

a) $\mu = E(X) = 0 \cdot 0{,}05 + 1 \cdot 0{,}20 + \cdots + 5 \cdot 0{,}05 = \underline{\underline{2{,}4}}$

b) $E(X^2) = 0^2 \cdot 0{,}05 + 1^2 \cdot 0{,}20 + \cdots + 5^2 \cdot 0{,}05 = 7{,}3$

$\sigma^2 = Var(X) = 7{,}3 - 2{,}4^2 = 1{,}54$

$\sigma = \sqrt{1{,}54} = \underline{\underline{1{,}241}}$

Chebyshev's Theorem:

The lower bound for probability of X being in the $k.\sigma$ interval around mean; which is the interval $(\mu - k.\sigma \, ; \mu + k.\sigma)$, is:

$$1 - \frac{1}{k^2}$$

Namely;

$$\boxed{P(\mu - k.\sigma \leq X \leq \mu + k.\sigma) \geq 1 - \frac{1}{k^2}}$$

Chebyshev's Theorem.

6.79 Use Chebyshev's theorem to determine the probability of getting a value

(a) within 4 standard deviations of the mean;

(b) within 2.5 standard deviations of the mean;

(c) within 2.8 standard deviations.

6.80 a) The number of CDs (compact disks) sold daily in a music store is a random variable with $\mu = 130$ and $\sigma = 10$. What does Chebyshev's theorem with $k = 3$ tell us about the number of CDs sold each day?

6.79) The corresponding probabilities are at least,

a) $1 - \frac{1}{4^2} = 0{,}9375$ b) $1 - \frac{1}{2{,}5^2} = 0{,}84$ c) $1 - \frac{1}{2{,}8^2} = 0{,}8724$

6.80) $\mu = 130$; $\sigma = 10$; $k = 3$

3σ interval around μ is;

$(\mu - 3\sigma \,;\, \mu + 3\sigma)$

$(130 - 3.10 \,;\, 130 + 3.10)$

$(100 \,;\, 160)$ and $1 - \frac{1}{k^2} = 1 - \frac{1}{3^2} \cong 0{,}89$

The probability that X (# of CD's sold daily) is in the interval (between) 100 and 160 is at least 0,89

6.80 b) At least what percentage of the CD's sold is between 113 and 147?

Ans: $(113 \,;\, 147)$

$147 = \mu + k.\sigma$

$147 = 130 + k.10$

$k = 1{,}7$

Min. prob. is

$1 - \frac{1}{1{,}7^2} = 0{,}654$

Example: 150 students entered to an exam. The mean of the exam is 68 and its variance is 20. At most how many students can have grades more than 79 or less than 57?

Ans: $n=150$

Min. Proportion of students in the interval (57;79):

$$(57;79) \longrightarrow \mu + k.\sigma = 79$$

$$68 + k.\sqrt{20} = 79$$

$$k = \frac{79-68}{\sqrt{20}} = 2,46$$

$$1 - \frac{1}{k^2} = 1 - \frac{1}{2,46^2} = 0,8347$$

At least $0,8347 . 150 = 125$ students are in the interval.

At most $150 - 125 = \underline{25}$ students are outside the interval.

IMPORTANT DISCRETE DISTRIBUTIONS

(I) The Binomial Distribution

$$X \sim \text{Binomial}(n;p)$$

where n: # of independent trials
p: Fixed probability of success
X: # of success

The pmf is, $P(X=x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$$\mu = E(X) = n.p \qquad \sigma^2 = Var(X) = n.p.(1-p)$$

Remember; $\binom{n}{x} = \dfrac{n!}{x!(n-x)!} = \dfrac{n.(n-1). \dots .(n-x+1)}{x!}$

6.8 A store advertised a clearance sale, and the probability is 0.20 that any one customer will make a purchase. Use Table I to determine the probability that in a random sample of eight shoppers

(a) three will make a purchase;

(b) two will make a purchase;

(c) one will make a purchase;

(d) none will make a purchase.

6.8) $n = 8$; $p = 0{,}20$

X: # of customers that make a purchase

$X \sim$ Binomial ($n = 8$; $p = 0{,}20$)

$f(x) = \binom{8}{x} 0{,}20^{x}\, 0{,}80^{8-x}$

a) $P(X=3) = \binom{8}{3} \cdot 0{,}20^{3} \cdot 0{,}80^{5} = 0{,}147$

b) $P(X=2) = \binom{8}{2} \cdot 0{,}20^{2} \cdot 0{,}80^{6} = 0{,}294$

c) $P(X=1) = \binom{8}{1} \cdot 0{,}20^{1} \cdot 0{,}80^{7} = 0{,}336$

d) $P(X=0) = \binom{8}{0} \cdot 0{,}20^{0} \cdot 0{,}80^{8} = 0{,}168$

n	x	...0,2
8	0	0,168
	1	0,336
	2	0,294
	3	0,147

These probabilities can also be found from Table I

6.15 A study conducted at a certain college shows that 60% of the school's graduates obtain a job in their chosen field within a year after graduation. Use Table I to find the probabilities that, within a year after graduation, among 14 randomly selected graduates of that college

(a) at least 6 will find a job in their chosen field;

(b) at most 3 will find a job in their chosen field;

(c) anywhere from 5 through 8 will find a job in their chosen field.

6.15) X: Number of school Graduates who find a job

$n = 14$; $p = 0{,}60$

$X \sim$ Binomial ($n = 14$; $p = 0{,}60$)

$f(x) = \binom{14}{x} \cdot 0{,}60^{x} \cdot 0{,}40^{14-x}$

a) $P(X \geq 6) = f(6) + f(7) + \dots + f(14)$

$= 0,092 + 0,157 + \dots + 0,001 = 0,941$

b) $P(X \leq 3) = f(0) + f(1) + f(2) + f(3) = 0 + 0 + 0,001 + 0,003 = 0,00$

c) $P(5 \leq X \leq 8) = f(5) + f(6) + f(7) + f(8)$

$= 0,061 + 0,092 + 0,157 + 0,207 = 0,497$

6.19 A food distributor claims that 80% of her 6-ounce cans of mixed nuts contain at least three pecans. To check on this, a consumer testing service decides to examine six of these 6-ounce cans of mixed nuts from a very large production lot and reject the claim if fewer than four of them contain at least three pecans. Use Table I to find the probabilities that the testing service will commit the error of

(a) rejecting the claim even though it is true;

(b) not rejecting the claim when in reality only 60% of the cans of mixed nuts contain at least three pecans;

(c) not rejecting the claim when in reality only 40% of the cans of mixed nuts contain at least three pecans.

6.19) X: # of 6-ounce cans that contain at least three pecans

a) $n = 6$; $p = 0,80$

$X \sim$ Binomial $(n = 6 ; p = 0,80)$ $\quad f(x) = \binom{6}{x} 0,80^{x} 0,20^{6-x}$

Reject the claim: $X \leq 4$

$P(X \geq 4) = 1 - P(X > 4) = 1 - [f(5) + f(6)]$

$= 1 - [0,393 + 0,262] = 0,345$

b) $X \sim$ Binomial $(n = 6 ; p = 0,60)$

Not Reject the claim: $X > 4$ $\quad f(x) = \binom{6}{x} 0,60^{x} 0,40^{6-x}$

$P(X > 4) = f(5) + f(6) = 0,187 + 0,047 = 0,234$

c) $X \sim$ Binomial $(n = 6 ; p = 0,40)$

Not Reject the claim: $X > 4$ $\quad f(x) = \binom{6}{x} 0,40^{x} 0,60^{6-x}$

$P(X > 4) = f(5) + f(6) = 0,037 + 0,004 = 0,041$

(II) The Hypergeometric Distribution

a: Type I objects
b: Non-Type I objects
Total: a+b objects

Select n objects Randomly
X: Number of type I objects

$$X \sim \text{Hypergeometric}(a;\, b;\, n)$$

$$P(X=x) = f(x) = \frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$$

$$\mu = E(X) = n \cdot \frac{a}{a+b} \qquad \sigma^2 = Var(X) = n \cdot \frac{a}{a+b} \cdot \frac{b}{a+b} \cdot \frac{a+b-1}{a+b-n}$$

6.26 Among 18 antique chairs for sale at an antique show, 15 are genuine and 3 are reproductions. If 3 chairs are randomly selected for consideration by a prospective purchaser, find the separate probabilities that these will include 0, 1, 2, or 3 reproductions.

6.26)

3 reproductions
15 genuine
Total = 18 Chairs

Select $n=3$ chairs
X: # of reproductions

$$X \sim \text{Hypergeometric}(a=3;\, b=15;\, n=3)$$

$$f(x) = \frac{\binom{3}{x}\binom{15}{3-x}}{\binom{18}{3}}$$

$$P(X=0) = f(0) = \frac{\binom{3}{0}\binom{15}{3}}{\binom{18}{3}} = 0{,}558 \;;\; P(X=1) = f(1) = \frac{\binom{3}{1}\binom{15}{2}}{\binom{18}{3}} = 0{,}386$$

$$P(X=2) = f(2) = \frac{\binom{3}{2}\binom{15}{1}}{\binom{18}{3}} = 0{,}055 \;;\; P(X=3) = f(3) = \frac{\binom{3}{3}\binom{15}{0}}{\binom{18}{3}} = 0{,}001$$

Alp Giray Özen | 0533 549 91 08 | alp@lecturemania.com | www.lecturemania.com

pmf of X is;

x	0	1	2	3
$f(x)$	0,558	0,386	0,055	0,001

6.29 Ten seedless grapefruit and eight seedy grapefruit were unintentionally mixed on a store display. If a clerk randomly selects four grapefruit from the display, what is the probability that he will select two seedless and two seedy grapefruit? Use the formula for the hypergeometric distribution and Table X, Binomial Coefficients, to solve this exercise.

6.29) $\boxed{\begin{array}{l}10 \text{ seedless} \\ 8 \text{ seedy}\end{array}}$ → select $n=4$

Total: 18 grapefruits

X: # of seedless grapefruits

$X \sim$ Hypergeometric $(a=10;\ b=8;\ n=4)$

$$f(x) = \frac{\binom{10}{x}\binom{8}{4-x}}{\binom{18}{4}}; \quad P(X=2) = f(2) = \frac{\binom{10}{2}\binom{8}{2}}{\binom{18}{4}} = \underline{\underline{0,412}}$$

6.36 Among the 120 employees of a company, 45 are members of a clerical workers union, and the others are members of the typesetters union. If 5 of the employees are chosen by lot to serve on a grievance committee, find the probability that 2 of them will be members of the clerical workers union and the other 3 members of the typesetters union, using

(a) the formula for the hypergeometric distribution;

(b) the binomial distribution as an approximation.

6.36) a) $\boxed{\begin{array}{l}45 \text{ clerical} \\ 75 \text{ typesetters}\end{array}}$ → Select $n=5$ employees

Total: 120 employees

X: # of clerical workers

$X \sim$ Hypergeometric $(a=45;\ b=75;\ n=5)$

$$f(x) = \frac{\binom{45}{x}\binom{75}{5-x}}{\binom{120}{5}}; \quad P(X=2) = f(2) = \frac{\binom{45}{2}\binom{75}{3}}{\binom{120}{5}} = 0,3508$$

b) $p = \frac{45}{120} = 0,375$; $X \sim$ Binomial $(n=5;\ p=0,375)$

$$f(x) = \binom{5}{x} 0{,}375^x \cdot 0{,}625^{5-x}; \quad P(X=2) = f(2) = \binom{5}{2} 0{,}375^2 \cdot 0{,}625^3 = 0{,}3433$$

(III) The Poisson Distribution

$X \sim Poisson(\lambda)$ λ: Rate or Mean

$$P(X=x) = f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$ x: # of success, λ: Average

$\mu = E(X) = \lambda$; $\sigma^2 = Var(X) = \lambda$

* Two types of questions

(i) Poisson distribution is mentioned OR Random Variable is Number of success and only average is known

(ii) Distribution is Binomial but n is high, p is small (preferably $n \cdot p \leq 7$), then set $\lambda = n \cdot p$ and use Poisson Approximation to Binomial Distribution.

6.43 According to the National Center for Health Statistics, U.S. Department of Health and Human Services, 1.2% of patients visited physicians' offices in a recent year because of nasal congestion. Use the Poisson approximation to the binomial distribution to determine the probability that in a random sample of 500 patients, 2 or 3 visited physicians' offices because of nasal congestion.

6.43) X: # of patients visited physicians' offices

$X \sim Binomial(n = 500 ; p = 0{,}012)$

Poisson Approximation: $\lambda = n \cdot p = 500 \cdot 0{,}012 = 6$

$X \underset{approx.}{\sim} Poisson(\lambda = 6)$

$$f(x) = \frac{e^{-6} \cdot 6^x}{x!}$$

$$P(X=2 \text{ OR } X=3) = f(2) + f(3) = \frac{e^{-6} \cdot 6^2}{2!} + \frac{e^{-6} \cdot 6^3}{3!} = \underline{0{,}134}$$

Alp Giray Özen | 0533 549 91 08 | alp@lecturemania.com | www.lecturemania.com

6.49 If a police department receives, on the average, $\lambda = 7$ reports of crimes per day, what is the probability that it will receive only 4 or 5 reports of crimes on a given day?

6.49) $X \sim Poisson(\lambda = 7)$

$$f(x) = \frac{e^{-7} \cdot 7^x}{x!}$$

$$P(X=4 \text{ OR } X=5) = f(4) + f(5) = \frac{e^{-7} \cdot 7^4}{4!} + \frac{e^{-7} \cdot 7^5}{5!} = 0{,}219$$

(IV) Multinomial Distribution

$$(X_1, X_2, \ldots, X_k) \sim Multinomial(p_1, p_2, \ldots, p_k, n)$$

$$P(X_1 = x_1, X_2 = x_2, \ldots, X_k = x_k) = \frac{n!}{x_1!\, x_2! \ldots x_k!} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdot \ldots \cdot p_k^{x_k}$$

where $x_1 + x_2 + \ldots + x_k = n$ and $p_1 + p_2 + \ldots + p_k = 1$

$E(X_i) = n \cdot p_i$; $Var(X_i) = n \cdot p_i \cdot (1 - p_i)$

It is like Binomial but we have more than two (success-failure) categories.

6.58 At the terminal of a certain airline, records disclose that 10% of their flights arrive early, 70% arrive on time, 12% arrive somewhat late, and 8% arrive very late. Use the formula for the multinomial distribution to determine the probability that in six randomly selected flights, one will arrive early, three will arrive on time, two will be somewhat late, and none will be very late.

X_1: Early flights ; X_2: On time flights ; X_3 : late flights,
X_4: Very late flights
$n = 6$

$$P(X_1 = 1, X_2 = 3, X_3 = 2, X_4 = 0)$$

$$= \frac{6!}{1!\,3!\,2!\,0!} \cdot 0{,}10^1 \cdot 0{,}70^3 \cdot 0{,}12^2 \cdot 0{,}08^0 = 0{,}0296$$

Continuous Probability Distributions

The random variable X which takes values on a real interval $\alpha < x < \beta$ is called a "Continuous Random Variable".

$f(x)$: Probability density function (pdf) of a continuous random variable is used to find the probability of X between two numbers. $f(x)$ is a valid pdf if;

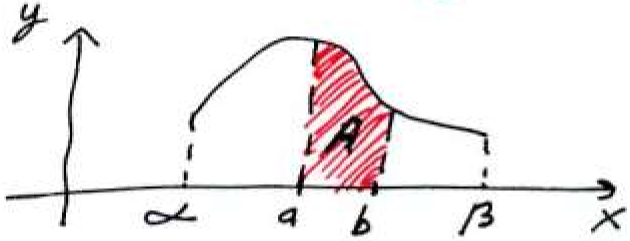
(i) $f(x) \geq 0$ → Nonnegative Probabilities

(ii) TOTAL AREA under $f(x)$ is equal to 1. → Total Probability is 1

we have;

$P(X = x) = 0$

$P(a < X < b) = A$

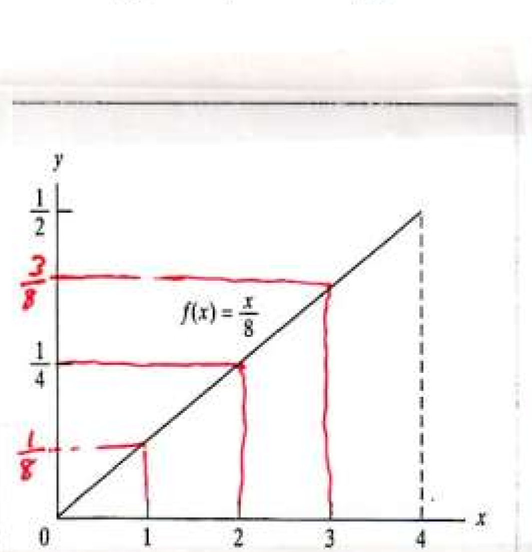


7.2 Suppose that a continuous random variable takes on values on the interval from 0 to 4 and that the graph of its probability density is given by the blue line of Figure 7.19.

(a) Verify that the total area under the curve is equal to 1.

(b) What is the probability that this random variable will take on a value less than 3?

(c) What is the probability that this random variable will take on a value between 1 and 2?



7.2) a) Total Area $= \dfrac{4 \cdot \frac{1}{2}}{2} = \dfrac{4}{4} = 1$

b) $P(X < 3) = \dfrac{3 \cdot \frac{3}{8}}{2} = \dfrac{9}{16}$

c) $P(1 < X < 2) = P(X < 2) - P(X < 1) = \dfrac{2 \cdot \frac{1}{4}}{2} - \dfrac{1 \cdot \frac{1}{8}}{2} = \dfrac{4}{16} - \dfrac{1}{16} = \dfrac{3}{16}$

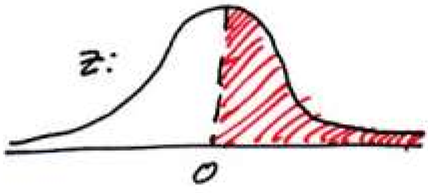
The Normal Distribution

$X \sim Normal(\mu ; \sigma^2)$

$E(X) = \mu \qquad Var(X) = \sigma^2$

The Standard Normal Distribution

$$Z = \frac{X-\mu}{\sigma} \quad : \quad Z \sim Normal(\mu = 0 ; \sigma^2 = 1^2)$$

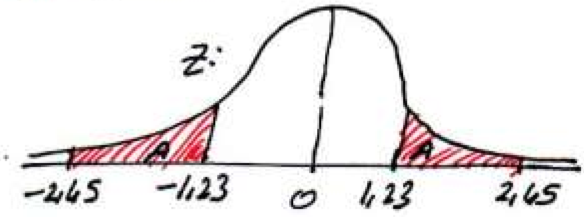


we have;

(i) Total AREA = 1

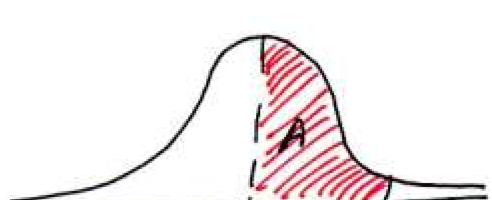
(ii) Half AREA = 0,5

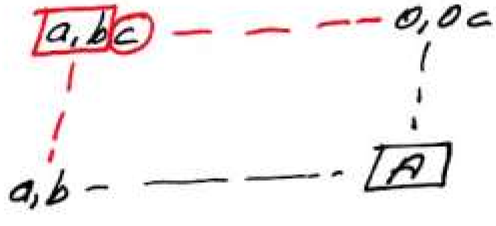
(iii) Symmetric AREA's are equal



$P(-2,45 < Z < -1,23) = P(1,23 < Z < 2,45)$

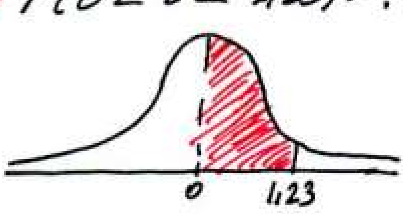
How to find probabilities from Z-table



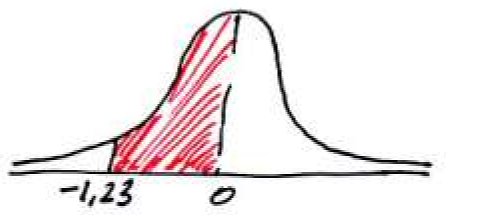


$A = P(0 < Z < a,bc)$

(I) $P(0 < Z < 1,23) = ?$



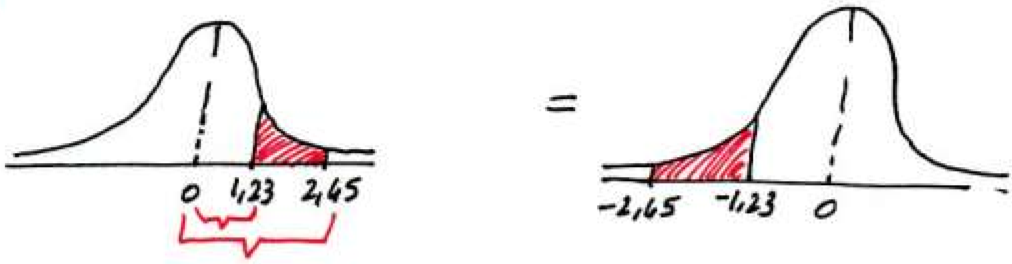
=



$P(0 < Z < 1,23) = P(-1,23 < Z < 0) = 0,3907$

Alp Giray Özen | 0533 549 91 08 | alp@lecturemania.com | www.lecturemania.com

(II) $P(1{,}23 < z < 2{,}45) = ?$



$P(1{,}23 < z < 2{,}45) = 0{,}4929 - 0{,}3907 = 0{,}1022 = P(-2{,}45 < z < -1{,}23)$

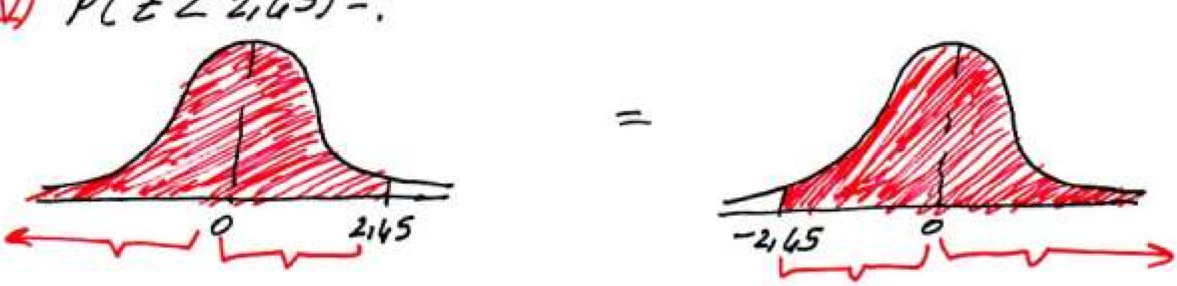
(III) $P(-1{,}23 < z < 2{,}45) = ?$

$P(-1{,}23 < z < 2{,}45) = 0{,}4929 + 0{,}3907 = 0{,}8836 = P(-2{,}45 < z < 1{,}23)$

(IV) $P(z > 1{,}23) = ?$

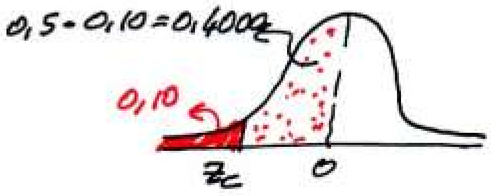
$P(z > 1{,}23) = 0{,}5 - 0{,}3907 = 0{,}1093 = P(z < -1{,}23)$

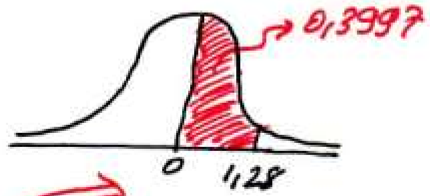
(V) $P(z < 2{,}45) = ?$



$P(z < 2{,}45) = 0{,}5 + 0{,}4929 = 0{,}9929 = P(z > -2{,}45)$

(VI) (i) $P(z < z_c) = 0{,}10 \Rightarrow z_c = ?$

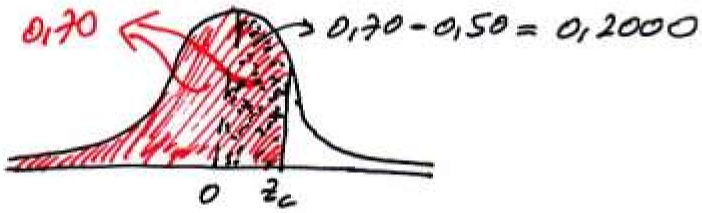




Nearest Number, $0{,}3997 \Rightarrow z_c = 1{,}28$

Negative Side $\Rightarrow \boxed{z_c = -1{,}28}$

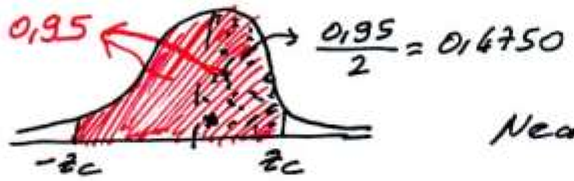
(ii) $P(z < z_c) = 0{,}70 \Rightarrow z_c = ?$



Nearest Number, $0{,}1985 \Rightarrow z_c = 0{,}52$

Positive Side $\Rightarrow \boxed{z_c = 0{,}52}$

(iii) $P(-z_c < z < z_c) = 0{,}95 \Rightarrow z_c = ?$



Nearest Number, $0{,}4750 \Rightarrow z_c = 1{,}96$

$\boxed{z_c = 1{,}96}$

* To find probabilities about $X \sim Normal(\mu; \sigma^2)$, we convert X to z by $\boxed{z = \frac{X-\mu}{\sigma}}$

* To find Numbers for X satisfying a probability, we find z_c and put in the equation $\boxed{X_c = \mu + z_c \cdot \sigma}$

7.12 Find z if

(a) the normal-curve area between 0 and z is 0.1915;

(b) the normal-curve area to the right of z is 0.8665;

(c) the normal-curve area to the right of z is 0.0228;

(d) the normal-curve area between $-z$ and z is 0.9500.

7.9 Find the area under the standard normal curve that lies

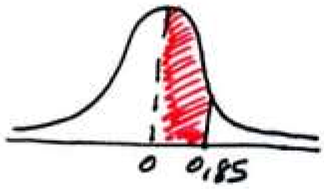
(a) between $z = 0$ and $z = 0.85$;

(b) to the right of $z = -0.65$;

(c) to the right of $z = 2.01$;

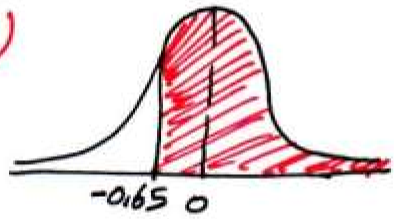
(d) between $z = -1.00$ and $z = -1.10$.

7.9) a)



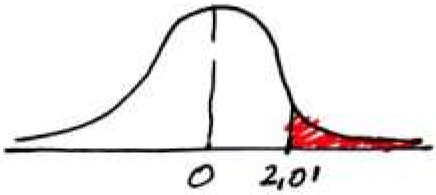
$P(0 < Z < 0{,}85) = 0{,}3023$

b)



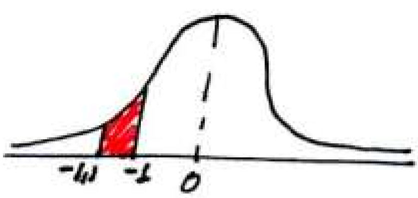
$P(Z > -0{,}65) = 0{,}5 + 0{,}2422 = 0{,}7422$

c)



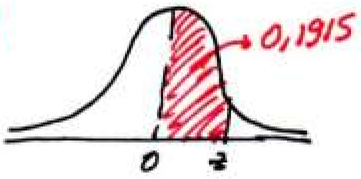
$P(Z > 2{,}01) = 0{,}5 - 0{,}4778 = 0{,}0222$

d)



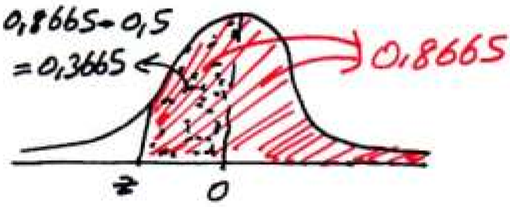
$P(-1{,}1 < Z < -1) = 0{,}3643 - 0{,}3413$
$= 0{,}0230$

7.12) a)



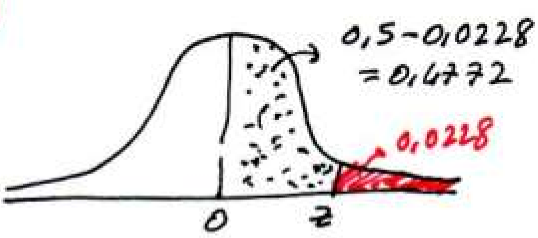
$P(0 < Z < z) = 0{,}1915 \Rightarrow z = 0{,}5$

b)



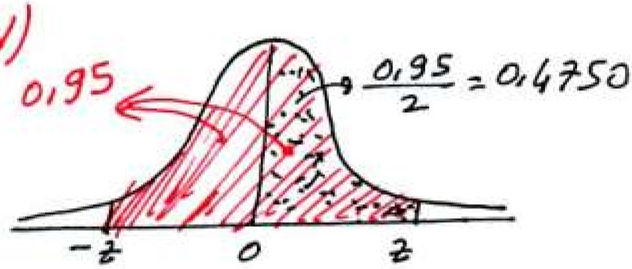
$P(z < Z < 0) = 0{,}3665 \Rightarrow z = -1{,}11$

c)



$P(0 < Z < z) = 0{,}4772 \Rightarrow z = 2$

d)



$P(0 < Z < z) = 0{,}4750 \Rightarrow z = 1{,}96$

7.15 A random value has a normal distribution with $\mu = 56.4$ and $\sigma = 4.8$. What are the probabilities that this random variable will take on a value

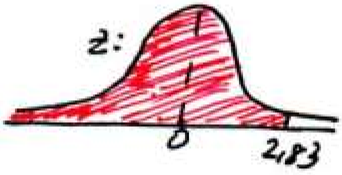
(a) less than 70.0;

(b) less than 50.0;

(c) between 50.0 and 70.0?

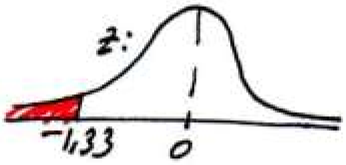
7.15) $X \sim Normal(\mu = 56{,}4 ;\ \sigma^2 = 4{,}8^2)$

a) $P(X < 70) = P\left(\frac{X-\mu}{\sigma} < \frac{70-56{,}4}{4{,}8}\right) = P(Z < 2{,}83)$



$= 0{,}5 + 0{,}4977 = 0{,}9977$

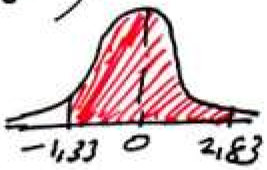
b) $P(X < 50) = P\left(\frac{X-\mu}{\sigma} < \frac{50-56{,}4}{4{,}8}\right) = P(Z < -1{,}33)$



$= 0{,}5 - 0{,}4082 = 0{,}0918$

c) $P(50 < X < 70) = P\left(\frac{50-56{,}4}{4{,}8} < \frac{X-\mu}{\sigma} < \frac{70-56{,}4}{4{,}8}\right)$

$= P(-1{,}33 < Z < 2{,}83) = 0{,}4082 + 0{,}4977 = 0{,}9059$

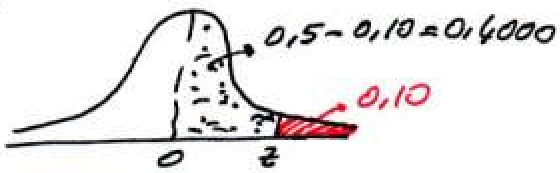


7.19 A normal distribution has the mean $\mu = 74.4$. Find its standard deviation if 10% of the area under the curve lies to the right of 100.0.

7.20 A random variable has a normal distribution with the standard deviation $\sigma = 10$. Find its mean if the probability is 0.8264 that it will take on a value less than 77.5.

7.21 For a certain random variable having the normal distribution, the probability is 0.33 that it will take on a value less than 245, and the probability is 0.48 that it will take on a value greater than 260. Find the mean and the standard deviation of this random variable.

7.19) $X \sim Normal(\mu = 74{,}4 ;\ \sigma^2)$



$P(X > 100) = 0{,}10$

$P\left(\frac{X-\mu}{\sigma} > \frac{100-74{,}4}{\sigma}\right) = 0{,}10$

$P\left(Z > \frac{100-74{,}4}{\sigma}\right) = 0{,}10$

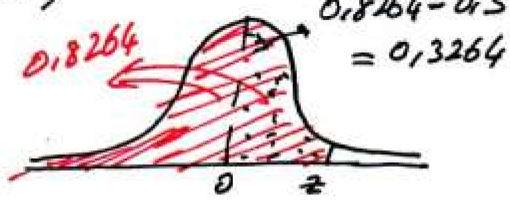
$z = 1{,}28$

$\frac{100-74{,}4}{\sigma} = 1{,}28$

$\sigma = \frac{100-74{,}4}{1{,}8} = 14{,}22$

7.20) $X \sim Normal(\mu ; \sigma^2 = 10^2)$

$P(X < 77,5) = 0,8264$



$P\left(\frac{X-\mu}{\sigma} < \frac{77,5-\mu}{10}\right) = 0,8264$

$z = 0,94$

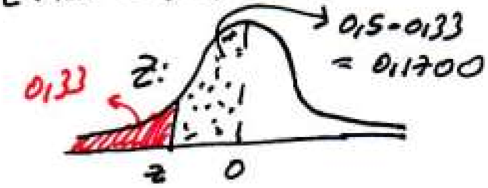
$P\left(Z < \frac{77,5-\mu}{10}\right) = 0,8264$

$\frac{77,5-\mu}{10} = 0,94$

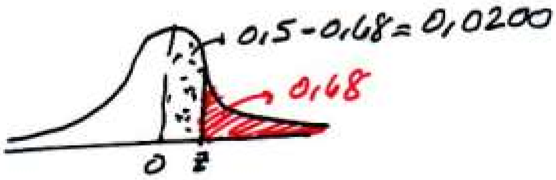
$\mu = 77,5 + 0,94.10 = 86,9$

7.21) $X \sim Normal(\mu ; \sigma^2)$

$P(X < 245) = 0,33$



$P(X > 260) = 0,48$



$z = 0,05$

$z = -0,44$

$\frac{260-\mu}{\sigma} = 0,05$

$\frac{245-\mu}{\sigma} = -0,44$

(ii) $\mu = 260 - 0,05.\sigma$

(i) $\mu = 245 + 0,44.\sigma$

(i) & (ii) $\Rightarrow$ $245 + 0,44\sigma = 260 - 0,05\sigma$

$0,49\sigma = 15 \Rightarrow \sigma = \frac{15}{0,49} = 30,61$

(i) $\Rightarrow \mu = 245 + 0,44.30,61 = 258,47$

Continuity Correction

If a discrete Random Variable is approximated by a Normal distribution, Replace

$P(X \geq k)$ with $P(X \geq k - 0,5)$ and

$P(X \leq k)$ with $P(X \leq k + 0,5)$

7.28 The management of a small ice skating rink knows that the daily numbers of skaters using the rink each morning is a random variable with a distribution that can be approximated closely by a normal distribution with the mean $\mu = 45.7$ and the standard deviation $\sigma = 4.8$. Using the continuity correction, determine the probability that the daily number of skaters using the rink during the morning session will

(a) exceed 50 on any given day;

(b) be less than 40 on any given day.

7.27 With reference to Exercise 7.28, above which ~~length~~ number lies the ~~longest~~ most 15% of the ~~weekend~~ skaters using the rink?

7.28) X: The daily number of skaters

$X \sim Normal(\mu = 45{,}7;\ \sigma^2 = 4{,}8^2)$

a) $P(X > 50) = P(X \geq 51) = P(X \geq 50{,}5) = P\left(\frac{X-\mu}{\sigma} \geq \frac{50{,}5-45{,}7}{4{,}8}\right)$

$= P(Z \geq 1) = 0{,}5 - 0{,}3413 = 0{,}1587$

7.28 b)

$P(X < 40) = P(X \leq 39)$

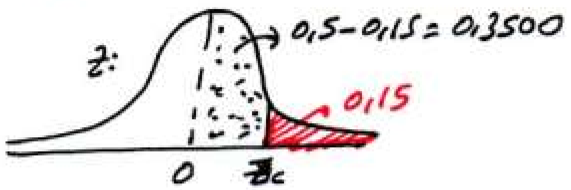
$= P(X \leq 39{,}5)$

$= P\left(Z \leq \frac{39{,}5 - 45{,}7}{4{,}8}\right)$

$= P(Z \leq -1{,}29) = 0{,}5 - 0{,}4015$

$= 0{,}0985$

7.27) $P(X > x_c) = 0{,}15$



$z_c = 1{,}04$

$Z = \frac{X-\mu}{\sigma}$

$1{,}04 = \frac{x_c - 45{,}7}{4{,}8}$

$x_c = 45{,}07 + 4{,}8 \cdot 1{,}04 = 50{,}69$

$\boxed{x_c \cong 51}$

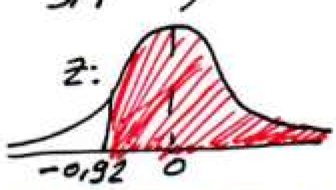
7.32 The annual number of tornadoes in a certain state is a random variable with $\mu = 28.2$ and $\sigma = 5.1$. Approximating the distribution of this random variable with a normal distribution, find the probability that there will be at least 24 tornadoes in the state in any given year.

7.32) X: The anual Number of tornadoes

$X \sim Normal(\mu = 28{,}2;\ \sigma^2 = 5{,}1)$

$P(X \geq 24) = P(X \geq 23{,}5) = P\left(\frac{X-\mu}{\sigma} \geq \frac{23{,}5 - 28{,}2}{5{,}1}\right)$

$= P(Z \geq -0{,}92) = 0{,}5 + 0{,}3212 = 0{,}8212$



Normal approximation to Binomial Distribution

Let, $X \sim$ Binomial $(n; p)$

If $np > 5$ and $n(1-p) > 5$, we can find Binomial Probabilities by Normal Distribution Letting

$$Y \sim \text{Normal}(\mu = n.p \,;\, \sigma^2 = n.p.(1-p))$$

Do NOT forget "Continuity Correction"

7.35 Check in each case whether the conditions for the normal approximation to the binomial distribution are satisfied.
(a) $n = 18$ and $p = 0.60$; (b) $n = 90$ and $p = .05$; (c) $n = 10$ and $p = 0.50$.

7.35) a) $np = 18.0{,}60 = 10{,}8$ ✓ b) $np = 90.0{,}05 = 4{,}5$ ✗ c) $np = 10.0{,}50 = 5$ ✗

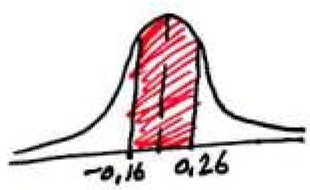
$n(1-p) = 18.0{,}40 = 7{,}2$ ✓ $n(1-p) = 90.0{,}95 = 85{,}5$ ✓ $n(1-p) = 10.0{,}50 = 5$ ✗

YES NO NO

7.37 If 62% of all clouds seeded with silver iodide show spectacular growth, what is the probability that among 24 clouds thus seeded exactly 15 will show spectacular growth? If the value given for this probability in the table of binomial probabilities is 0.1661, what is the error of the approximation?

7.38 A dating service finds that 15% of the couples that it matches eventually get married. In the next 50 matches that the service makes, find the probabilities that
(a) at least 6 couples marry; (b) at most 10 couples marry.

7.39 The manager of an air-conditioning company knows that the number of service calls made each day is a random variable having approximately a normal distribution with the mean 34.9 service calls, and the standard deviation 4.8 calls. What are the probabilities that in any given day the company will make
(a) exactly 31 air-conditioning service calls;
(b) at most 31 air-conditioning service calls?



7.37) $X \sim$ Binomial $(n = 24 \,;\, p = 0{,}62)$ $np = 24.0{,}62 = 14{,}88$ ✓
$n(1-p) = 24.0{,}38 = 9{,}12$ ✓

$Y \sim \text{Normal}(\mu = 24.0{,}62 = 14{,}88 \,;\, \sigma^2 = 24.0{,}62.0{,}38 = 5{,}65)$

$\sigma = \sqrt{5{,}65} = 2{,}38$

$$P(X=15) = P(14{,}5 < Y < 15{,}5) = P\left(\frac{14{,}5 - 14{,}88}{2{,}38} < Z < \frac{15{,}5 - 14{,}88}{2{,}38}\right) = P(-0{,}16 < Z < 0{,}26)$$
$$= 0{,}0636 + 0{,}1026$$
$$= 0{,}1662$$

Alp Giray Özen | 0533 549 91 08 | alp@lecturemania.com | www.lecturemania.com

7.38) $X \sim \text{Binomial}(n=50; p=0{,}15)$ $\quad np = 50 \cdot 0{,}15 = 7{,}5$ ✓ $\quad n(1-p) = 50 \cdot 0{,}85 = 42{,}5$ ✓

$$\mu = np = 50 \cdot 0{,}15 = 7{,}5$$

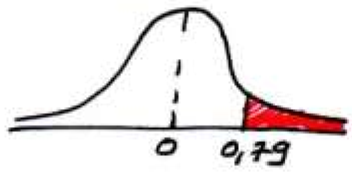
$$\sigma^2 = n \cdot p(1-p) = 50 \cdot 0{,}15 \cdot 0{,}85 = 6{,}375$$

$$\sigma = \sqrt{6{,}375} = 2{,}525$$

$$Y \sim \text{Normal}(\mu = 7{,}5 ; \sigma^2 = 2{,}525^2)$$

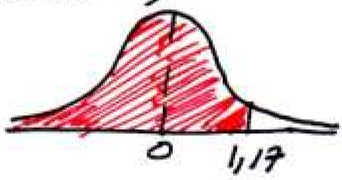
a) $P(X \geq 6) = P(Y \geq 5{,}5) = P\left(\frac{Y-\mu}{\sigma} \geq \frac{5{,}5 - 7{,}5}{2{,}525}\right)$

$= P(Z \geq 0{,}79) = 0{,}5 - 0{,}2852 = 0{,}2148$



b) $P(X \leq 10) = P(Y \leq 10{,}5) = P\left(\frac{Y-\mu}{\sigma} \leq \frac{10{,}5 - 7{,}5}{2{,}525}\right)$

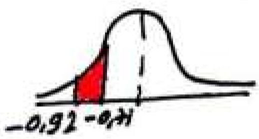
$= P(Z \leq 1{,}17) = 0{,}5 + 0{,}3790 = 0{,}8790$



7.39)

$$X \sim \text{Normal}(\mu = 34{,}9 ; \sigma^2 = 4{,}8^2)$$

a) $P(X=31) = P(30{,}5 < X < 31{,}5) = P\left(\frac{30{,}5 - 34{,}9}{4{,}8} < Z < \frac{31{,}5 - 34{,}9}{4{,}8}\right)$



$= P(-0{,}92 < Z < -0{,}71) = 0{,}3212 - 0{,}2611 = 0{,}0681$

b) $P(X \leq 31) = P(X \leq 31{,}5) = P(Z \leq -0{,}71) = 0{,}5 - 0{,}2611$

$= 0{,}2389$