STATISTICS for LAWYERS & SOCIAL SCIENCES | CHAPTERS 8, 9 & 10

## Simple Random Sampling:

A (simple) random sample from a finite population size $N$ is a sampling procedure such that all possible samples of size $n$ have equal probability to be selected. A random sample of size $n$ can be selected from $N$ units in $\binom{N}{n}$ different ways.

**8.5** What is the probability of each possible sample if a random sample of size 3 is to be drawn from a finite population of size

(a) 4;

(b) 6;

(c) 12;

(d) 18?

**8.6** What is the probability of each possible sample of size 4 being drawn from a finite population of size 18?

8.5) $n=3$ ; (a) $N=4 \Rightarrow \binom{4}{3}=1$ (b) $N=6 \Rightarrow \binom{6}{3}=20$

(c) $N=12 \Rightarrow \binom{12}{3}=220$ (d) $N=18 \Rightarrow \binom{18}{3}=816$

8.6) Number of Samples $=\binom{N}{n} \Rightarrow$ Prob. of each sample $= \dfrac{1}{\binom{N}{n}}$

Then; Prob. of each sample $= \dfrac{1}{\binom{18}{4}} = \dfrac{1}{3060} = 0{,}000327$

**8.8** List the $\binom{5}{2}=10$ possible samples of size $n=2$ that can be drawn from the finite population whose elements are denoted by the letters $a, b, c, d$, and $e$.

8.8) Population: $\{a, b, c, d, e\}$ ; select $n=2$ items.

Possible samples: $\{\{a,b\}, \{a,c\}, \{a,d\}, \{a,e\},$
$\{b,c\}, \{b,d\}, \{b,e\},$
$\{c,d\}, \{c,e\},$
$\{d,e\}\}$

**8.15** Randy and Susan are both members of a population of 60 students. A researcher is going to select 10 students from this population.

(a) What is the probability that Randy will be in the sample?

(b) What is the probability that Susan will be in the sample?

(c) What is the probability that both will be in the sample?

(d) Is your solution to (c) greater than or less than $\left(\frac{10}{60}\right)^2 = \frac{1}{36}$?

8.15) $N=60$ ; $n=10$ students selected

a) Sample: { Randy + 9 other students } $\Rightarrow \binom{59}{9}$ ways.

Random Sample of size 10 $\Rightarrow \binom{60}{10}$ ways.

$$P(\text{Randy in the Sample}) = \frac{\binom{59}{9}}{\binom{60}{10}} = \frac{10}{60} = 0,167$$

b) $P(\text{Susan in the Sample}) = 0,167$

c) Sample: { Randy, Susan + 8 other Students } $\Rightarrow \binom{58}{8}$ ways

$$P(\text{Both in the Sample}) = \frac{\binom{58}{8}}{\binom{60}{10}} = \frac{10 \cdot 9}{60 \cdot 59} = 0,0254$$

d) $\frac{10 \cdot 9}{60 \cdot 59} = \left(\frac{10}{60}\right) \cdot \left(\frac{9}{59}\right) < \frac{10}{60} \cdot \frac{10}{60} = \left(\frac{10}{60}\right)^2 \Rightarrow$ Less Than!

## Sampling Distribution

The probability distribution of a sample statistics such as sample mean, sample variance ...etc, is called "Sampling Distribution". A sampling distribution is found as follows:

(i) Obtain Each Possible sample
(ii) Find the relevent statistics for each sample point.
(iii) Assign probabilities to the values of statistics.

**8.18** Random samples of size $n = 2$ are drawn from a finite population that consists of the numbers 2, 4, 6, and 8.

(a) Calculate the mean and the standard deviation of this population.

(b) List the six possible random samples of size $n = 2$ that can be drawn from this population and calculate their means.

(c) Use the results of part (b) to construct the sampling distribution of the mean for random samples of size $n = 2$ from the given population.

(d) Calculate the mean and standard deviation of the sampling distribution obtained in part (c) and verify the result by substituting $n = 2$, $N = 4$, and the value of $\sigma$ obtained in part (a) into the second of the two standard error formulas on page 275.

8.18) a) Population: $\{2, 4, 6, 8\}$ : $N = 4$

$$\mu = \frac{\sum x_i}{N} = \frac{2+4+6+8}{4} = 5$$

$$\sigma^2 = \frac{\sum (x_i-\mu)^2}{N} = \frac{(2-5)^2+(4-5)^2+(6-5)^2+(8-5)^2}{4} = \frac{20}{4} = 5$$

b)

| Sample | 2,4 | 2,6 | 2,8 | 4,6 | 4,8 | 6,8 |
|---|---|---|---|---|---|---|
| mean | 3 | 4 | 5 | 5 | 6 | 7 |
| | $\frac{2+4}{2}$ | $\frac{2+6}{2}$ | $\frac{2+8}{2}$ | $\frac{4+6}{2}$ | $\frac{4+8}{2}$ | $\frac{6+8}{2}$ |

c)

| $\bar{X}$ | 3 | 4 | 5 | 6 | 7 |
| --- | --- | --- | --- | --- | --- |
| $p(\bar{x})$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

d) $\mu_{\bar{x}} = E(\bar{X}) = \sum \bar{x} \cdot p(\bar{x}) = 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{2}{6} + 6 \cdot \frac{1}{6} + 7 \cdot \frac{1}{6} = 5 = \mu$

$E(\bar{X}^2) = \sum \bar{x}^2 \cdot p(\bar{x}) = 3^2 \cdot \left(\frac{1}{6}\right) + 4^2 \cdot \left(\frac{1}{6}\right) + 5^2 \cdot \frac{2}{6} + 6^2 \cdot \frac{1}{6} + 7^2 \cdot \frac{1}{6} = \frac{80}{3}$

$\sigma_{\bar{x}}^2 = Var(\bar{X}) = E(\bar{X}^2) - \mu_{\bar{x}}^2 = \frac{80}{3} - 5^2 = \frac{5}{3} = \underbrace{\frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}}_{\frac{5}{2} \cdot \frac{4-2}{4-1}}$

Sampling Distribution of the Mean;

Summarizing the results we obtained from Exercise 8.18, The mean and variance of a mean from a finite population is;

$$\boxed{\mu_{\bar{x}} = \mu} \quad \text{and} \quad \boxed{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}}$$

So, the standard deviation is; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

where; $\sqrt{\frac{N-n}{N-1}}$ is finite population correction factor

For infinite samples, we have:

$$\boxed{\mu_{\bar{x}} = \mu} \quad \text{and} \quad \boxed{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}}$$

We omit finite population correction factor if $n$ is less than 5% of population size $N$.

**8.23** Find the value of the finite population correction factor for $n = 100$ and $N = 10,000$. Would you use this correction factor in a real problem requiring the standard error of the mean of a finite population? Why or why not?

8.23) $\frac{n}{N} = \frac{100}{10\,000} = 0{,}01 < 0{,}05$ so, we may omit finite population correction factor.

* Note that, standard deviation of sampling distribution of $\bar{X}$ : $\sigma_{\bar{X}}$ is called Standard Error of the Mean.

Central Limit Theorem

Remember, by Chebyshev's theorem, the probability of $\bar{X}$ to be in $k.\sigma_{\bar{X}}$ interval around $\mu$ is at least $1-\frac{1}{k^2}$. Namely,

$$P(\mu - k.\sigma_{\bar{X}} \leq \bar{X} \leq \mu + k.\sigma_{\bar{X}}) \geq 1-\frac{1}{k^2}$$

The "Central Limit Theorem" asserts that for any distribution, sampling distribution of the mean for large $n$ $(n \geq 30)$ is approximately Normal. Then, we can find probabilities for $\bar{X}$ using $Z$ table.

Remember, if $X \sim Normal(\mu ; \sigma^2)$, we had

$$\boxed{Z = \frac{X-\mu}{\sigma}}.$$

Then, for large $n$, $\bar{X} \sim Normal(\mu_{\bar{X}} = \mu ; \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n})$

so; $$\boxed{Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}$$



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**8.33** A commuting student wants to use a sample of 36 driving times to estimate the mean time it takes to drive to the college each morning. If the standard deviation of the time required to make the trip is assumed to be 11 minutes, what can the student assert about the probability that his error will be less than 3.3 minutes if he uses

(a) Chebyshev's theorem;

(b) the central limit theorem?

8.33) Interval of $\bar{X} \in (\mu \pm \underbrace{k \cdot \sigma_{\bar{x}}}_{\text{Error } (E)})$

$n = 36$ ; $\sigma = 11$ ; $E = 3,3$

$$E = k \cdot \sigma_{\bar{x}}$$

$$3,3 = k \cdot \frac{11}{\sqrt{36}} \Rightarrow k = \frac{3,3 \cdot \sqrt{36}}{11} = 1,8$$

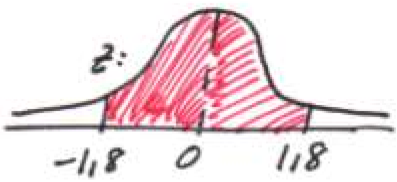
a) By Chebyshev's Thm, his error probability is at least $1 - \frac{1}{1,8^2} = 0,6914$

b) By Central Limit Theorem; The probability is approximately

$$P(\mu - 3,3 < \bar{X} < \mu + 3,3) = P(-3,3 < \bar{X} - \mu < 3,3)$$

$$= P\left(\frac{-3,3}{11/\sqrt{36}} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{3,3}{11/\sqrt{36}}\right) = P(-1,8 < z < 1,8)$$

$$= 2 \cdot 0,4713 = 0,9426$$



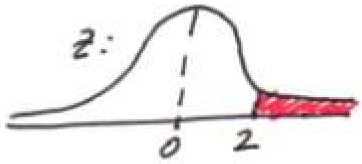

**8.38** During the last week of the semester, students at a certain college spend on the average 4.2 hours using the school's computer terminals with a standard deviation of 1.8 hours. For a random sample of 36 students at that college, find the probabilities that the average time spent using the computer terminals during the last week of the semester is

(a) at least 4.8 hours;

(b) between 4.1 and 4.5 hours.

8.38) $\mu = 4,2$ ; $\sigma^2 = 1,8$ ; $n = 36$

a) $P(\bar{X} > 4,8) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{4,8-4,2}{1,8/\sqrt{36}}\right) = P(Z > 2)$




$= 0,5 - 0,4772 = 0,0228$

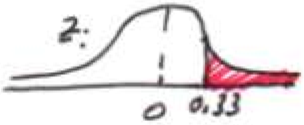
b) $P(4,1 < \bar{X} < 4,5) = P\left(\frac{4,1-4,8}{1,8/\sqrt{36}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{4,5-4,8}{1,8/\sqrt{36}}\right)$

$= P(-2,33 < Z < -1) = 0,4901 - 0,3413 = 0,1488$


c) If the time spent has a Normal Distribution, find the probabilities at (a) and (b) for a (single) student's time spent using the school's computer terminals.

Answer

$P(X > 4,8) = P\left(\frac{X-\mu}{\sigma} > \frac{4,8-4,2}{1,8}\right) = P(Z > 0,33$




$= 0,5 - 0,1293 = 0,3707$

$P(4,1 < X < 4,5) = P\left(\frac{4,1-4,8}{1,8} < \frac{X-\mu}{\sigma} < \frac{4,5-4,8}{1,8}\right)$

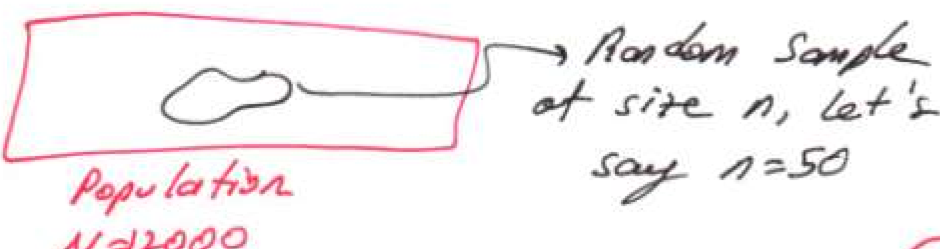
$= P(-0,39 < Z < -0,33) = 0,1517 - 0,1293 = 0,0224$

## Population versus Sample

Population consists of ALL the units that we want to make inference. For example, if we want to study mean Food Expenditure of Bilkent University, population is ALL the students at Bilkent, let's say $N=12000$.

A random variable $X$ is considered here as a student's weekly food expenditure. Note that it is random since it varies over students.

| | (Unknown Constants) Population Parameters | | (Known Variables) Sample statistics |
|---|---|---|---|
| MEAN | $\mu$ | INFERENCE ⟵ | $\bar{X} = \dfrac{\sum X_i}{n}$ |
| VARIANCE | $\sigma^2$ | (i) Confidence Interval | $s^2 = \dfrac{\sum X_i^2 - \dfrac{(\sum X_i)^2}{n}}{n-1}$ |
| PROPORTION | $p$ | (ii) Hypothesis Testing | $\hat{p} = \dfrac{\sum y_i}{n} : y_i = \begin{cases} 1 & \text{success} \\ 0 & \text{failure} \end{cases}$ |



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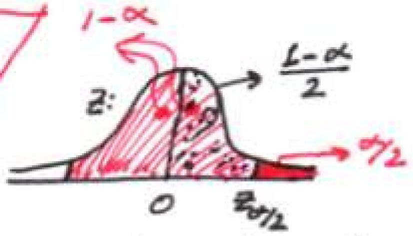
# Confidence Invervals

## (I) for population mean: $\mu$ (when $n$ is large: $n \geq 30$)

$(1-\alpha).100\%$ confidence interval for $\mu$ is;

$$\boxed{\overline{X} \pm \underbrace{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{=E} \quad : \quad \overline{X} \pm E}$$




$\boxed{E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$ is called (maximum) error of estimate.

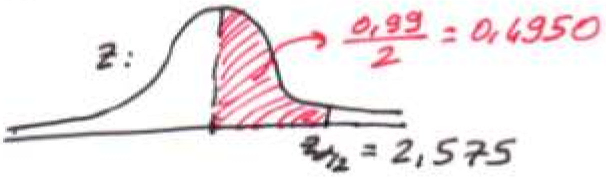
Given $E$, the required sample size is found by;

$$\boxed{n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 \text{: round up!}}$$

**9.3** A specialist in precision machinery wants to estimate the mean expansion of certain pistons (in inches) on the basis of a sample of 42 of these pistons. The expansion is caused by the heat generated after the engines have been started, and it is known that $\sigma = 0.020$ inch. If the specialist considers the data to constitute a random sample, what can he assert with probability 0.99 about the maximum error if he uses the mean of his sample as an estimate of the actual mean expansion of such pistons.

**9.4** A buyer for a large wholesaler of vegetables plans to estimate the mean weight of the Hubbard squash in a freight car load of these squash. To obtain this estimate the buyer will determine the weight of a gross (144) of the squash from this shipment. The buyer knows, from experience and from inspection, that the standard deviation is $\sigma = 3.00$ pounds. What can the buyer assert, with probability 0.95, about the maximum error of the estimate?

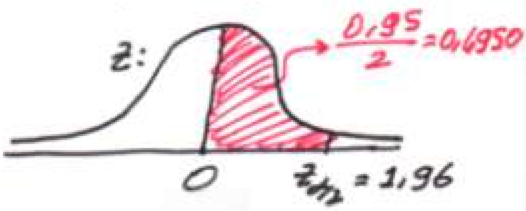
9.3) $n = 42$; $\sigma = 0{,}020$; $1-\alpha = 0{,}99$; $E = ?$




$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2{,}575 \cdot \frac{0{,}020}{\sqrt{42}} = \underline{0{,}008}$$

9.4) $n = 144$ ; $\sigma = 3$ ; ~~[crossed out]~~ $1-\alpha = 0{,}95$ ; $E = ?$

$$E = 1{,}96 \cdot \frac{3}{\sqrt{144}} = 0{,}49$$

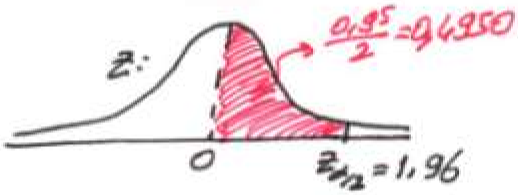



**9.13** The marketing manager of a toy company needs to know, with reasonable accuracy, how long it takes a child of a certain age to assemble a model airplane. If preliminary studies have shown that the standard deviation is 15 minutes, how large a sample will the manager need to be able to assert with a probability of 0.95 that the sample mean will be off by at most 3 minutes?

**9.14** A park ranger wants to know the average size of trout taken from a certain lake. How large a sample of trout must be taken to be able to assert with a probability of 0.98 that a sample mean will not be off by more than 0.5 inch? Assume that it is known from previous studies that $\sigma = 2.5$ inches.

9.13) $\sigma = 15$ ; $1-\alpha = 0{,}95$ ; $E = 3$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1{,}96 \cdot 15}{3}\right)^2 = 96{,}04 \overset{\text{round up!}}{=} 97$$




9.14) $1-\alpha = 0{,}98$ ; $E = 0{,}5$ ; $\sigma = 2{,}5$

$$n = \left(\frac{2{,}33 \cdot 2{,}5}{0{,}5}\right)^2 = 135{,}7 \overset{\text{round up!}}{=} 136$$

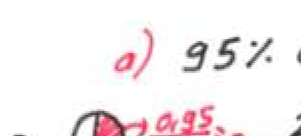

**9.19** A study of a chemical factory's discharge of pollutants over 49 randomly selected days showed that the mean daily discharge was $\bar{x} = 25.1$ tons with standard deviation $s = 8.5$ tons. Construct a confidence interval for the true mean of the factory's discharge of contaminants

(a) using a 0.95 degree of confidence;

(b) using a 0.99 degree of confidence.

9.19) $n = 49$ ; $\bar{X} = 25{,}1$ ; $s = 8{,}5 = \sigma$ (since $n$ is large)

a) 95% C.I. for $\mu$ is;

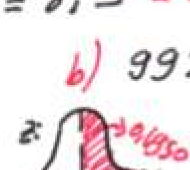



$$25{,}1 \pm 1{,}96 \cdot \frac{8{,}5}{\sqrt{49}}$$

$-$ $+$

$(22{,}72 \, ; \, 27{,}48)$

b) 99% C.I. for $\mu$ is;




$$\bar{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$25{,}1 \pm 2{,}575 \cdot \frac{8{,}5}{\sqrt{49}}$$

$-$ $+$

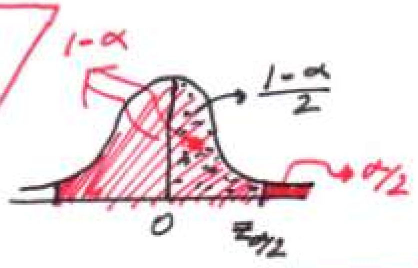
$(21{,}97 \, ; \, 28{,}23)$

* C.I. is wider when $(1-\alpha)$ is higher!

(II) for population proportion: $p$

$(1-\alpha).100\%$ Confidence interval for $p$ is;

$$\boxed{\hat{p} \pm \underbrace{z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}.(1-\hat{p})}{n}}}_{=E} \;:\; \hat{p} \pm E}$$




$$\boxed{E = z_{\alpha/2}\cdot\sqrt{\frac{p\,(1-p)}{n}}}$$ and $$\boxed{\hat{p} = \frac{x}{n}\ ;\ x: \#\text{ of success}}$$

Here, we have some points to consider. What is $p$ in the formula of $E$ (also in the formula of $n$ that will be given)?

(i) If there is an estimate $\hat{p}$; replace $p$ with $\hat{p}$

(ii) If there is an interval estimate for $p$, take $p$ the value closer to $0,5$. For example, if $p \in (0,1;0,4)$ take $p=0,4$ and if $p \in (0,7;0,8)$ take $p=0,7$

(iii) If there is no information $p$ apriori, take $p=0,5$.

Given $E$, the required sample size is found by

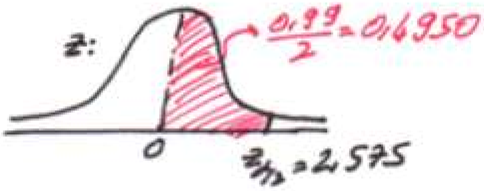
$$\boxed{n = p.(1-p).\left[\frac{z_{\alpha/2}}{E}\right]^2 : \text{round up!}}$$

**9.44** Among 100 randomly selected voters in a certain town, 30 were opposed to floating a bond issue to build a new school building. If we use the sample proportion $\frac{30}{100} = 0.30$ to estimate the corresponding true proportion, what can we assert with 99% confidence about the maximum error?

**9.45** With reference to Exercise 9.44, construct a 95% confidence interval for the true proportion of voters in the town who are opposed to floating a bond issue to build a new school building.

9.44) $n = 100 ; X = 30 ; \hat{p} = \frac{30}{100} = 0,30 ; 1-\alpha = 0,99 ; E = ?$

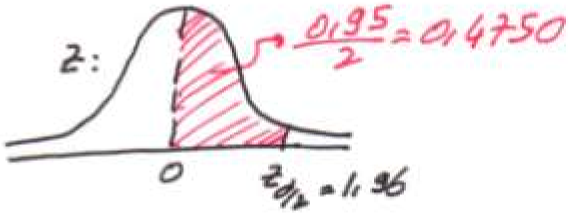
$$E = z_{\alpha/2} \cdot \sqrt{\frac{p(1-p)}{n}} = 2,575 \cdot \sqrt{\frac{0,30 \cdot 0,70}{30}} = 0,215 \Longrightarrow$$




9.45) 95% C.I. for p is;

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0,3 \pm 1,96 \cdot \sqrt{\frac{0,3 \cdot 0,7}{30}}$$

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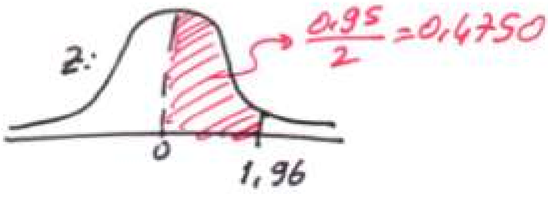
$(0,136 ; 0,464)$

**9.50** In a sample survey of 2,500 commuters, 900 said that they regularly use public transportation. Construct a 95% confidence interval for the corresponding true proportion of commuters who regularly use public transportation.

9.50) $n = 2500 ; X = 900 ; \hat{p} = \frac{900}{2500} = 0,36 ; 1-\alpha = 0,95$

95% C.I. for p is;

$$0,36 \pm 1,96 \cdot \sqrt{\frac{0,36 \cdot 0,64}{2500}}$$

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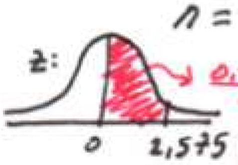
$(0,341 ; 0,379)$

**9.54** A private opinion poll is engaged by a politician to estimate what proportion of her constituents favor the decriminalization of certain narcotics violations. How large a sample will the poll have to take to be able to assert with probability 0.99 that the sample proportion will be off by no more than 0.02?

9.54) $1-\alpha = 0{,}99$ ; $E = 0{,}02$ ; $p = 0{,}5$ → Since there's NO information about $p$

$$n = p\cdot(1-p)\cdot\left[\frac{z_{\alpha/2}}{E}\right]^2 = 0{,}5\cdot 0{,}5\cdot\left[\frac{2{,}575}{0{,}02}\right]^2 = 4144{,}2 \overset{\text{round up!}}{=} 4145$$

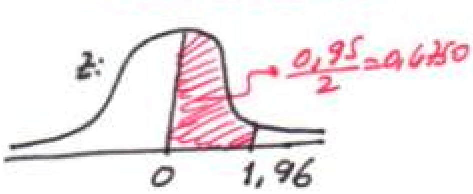



**9.56** A large personal computer manufacturer wants to determine from a sample what proportion of households intend to purchase personal computers within the next 12 months. How large a sample will the manufacturer need to be able to assert with a probability of at least 0.95 that the sample proportion will not differ from the true proportion by more than 0.05?

**9.57** With reference to Exercise 9.56, how large a sample will the manufacturer need to be able to assert with probability 0.95 that the sample proportion will be off by no more than 0.05, if he has good reason to believe that the true proportion is somewhere between 0.10 and 0.20?

9.56) $1-\alpha = 0{,}95$ ; $E = 0{,}05$ ; $p = 0{,}5$ → Since there is NO information about $p$

$$n = 0{,}5\cdot 0{,}5\cdot\left[\frac{1{,}96}{0{,}05}\right]^2 = 384{,}2 \overset{\text{round up!}}{=} 385$$




9.57) $1-\alpha = 0{,}95$; $E = 0{,}05$; $p = 0{,}2$ → Since $p \in (0{,}1;\ \underbrace{0{,}2}_{\text{closer to } 0{,}5})$

$$n = 0{,}2\cdot 0{,}8\cdot\left[\frac{1{,}96}{0{,}05}\right]^2 = 245{,}86 \overset{\text{round up!}}{=} 246$$

# HYPOTHESIS TESTING

* Hypothesis Testing Questions are YES/NO questions
"Can we conclude that...?"
"Is there sufficient evidence that...?"
"Is the claim true?"... etc.

Example: I want to open a restaurant at Bilkent. I think that opening the restaurant will be profitable if average food expenditure of the students is more than 100TL. Of a random sample of 50 students, mean food expenditure is found to be 103,76TL with variance 210. At 5% significance level, should I open the restaurant?

Answer: HYPOTHESIS TESTING STEPS:

(i) $H_0$, $H_A$ and $\alpha$ → State what to test
(ii) Test Statistics → what is the formula?
(iii) Decision Criteria → When to "Reject $H_0$"
(iv) Calculation → Calculate test statistics
(v) Decision & Conclusion → "Reject $H_0$" OR "Do NOT Reject $H_0$"

These steps are performed as follows:

(i) $H_0$, $H_A$ and $\alpha$ → $H_0$ and $H_A$ are complementary

$H_0: \mu \leq 100$

$H_A: \mu > 100$ → Inequality is always at $H_A$

$\alpha = 0{,}05$

(ii) Test Statistics (One sample Mean test, Large Sample)

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$



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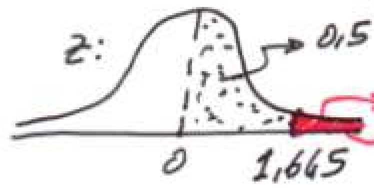
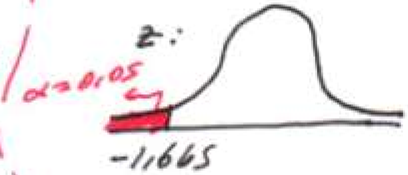
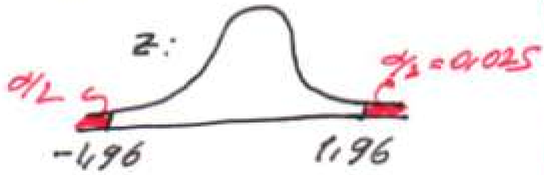
(iii) Decision Criteria




→ Rejection Region, same side with $H_A$ (we have >)

Reject $H_0$ if $Z > 1,645$

* Rejection Region: Shade α as being with the same side of $H_A$. Consider the following tests:

| | | (This is called a 2-sided test) |
|---|---|---|
| $H_0: \mu \leq 100$ | $H_0: \mu \geq 70$ | $H_0: \mu = 18$ |
| $H_A: \mu > 100$ | $H_A: \mu < 70$ | $H_A: \mu \neq 18$ |
| $\alpha = 0,05$ | $\alpha = 0,05$ | $\alpha = 0,05$ |
|   |   |   |
| Reject $H_0$ if $Z > 1,665$ | Reject $H_0$ if $Z < -1,665$ | Reject $H_0$ if $Z > 1,96$ OR $Z < -1,96$ |

(iv) Calculation

$$\left.\begin{array}{l} n = 50 \\ \bar{X} = 103,76 \\ \sigma^2 = 210 \end{array}\right\} \Rightarrow \quad Z = \frac{103,76}{\sqrt{210/50}} = 1,835$$

(v) Decision and Conclusion

1,835 > 1,645 so we Reject $H_0$. I can conclude that mean food expenditure is more than 100TL and open the restaurant at $\alpha = 0,05$.



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**10.15** The manager of the women's dress department of a department store wants to know whether the true average number of women's dresses sold per day is 24. If in a random sample of 36 days the average number of dresses sold is 23 with a standard deviation of seven dresses, is there, at the 0.05 level of significance, sufficient evidence to reject the null hypothesis that $\mu = 24$?

**10.16** The nurse in a dentist's office claims that he treats 20 patients per day. To verify this claim, the nurse randomly selects a sample of records for 100 days and concludes that the mean number of patients is 18.1 with a standard deviation of 5.4. Use the 0.05 level of significance to test the null hypothesis $\mu = 20$ patients against the alternative hypothesis $\mu < 20$ patients.

**10.17** While investigating a claim that a coin-operated hot coffee machine is dispensing too much hot coffee into hand-held cups (creating a dangerous situation for holders of cups), the owner of the machine takes a random sample of 49 "8-ounce servings" from a large number of servings and finds that the mean is 8.2 fluid ounces with a standard deviation of 0.4 fluid ounces. Is this evidence of overfilling at the 0.01 level of significance?

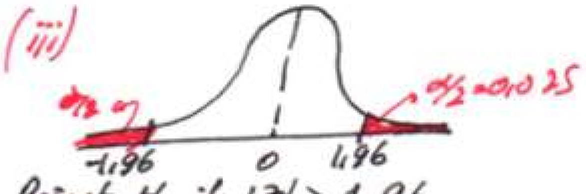
10.15) (i) $H_0: \mu = 24$

$H_A: \mu \neq 24$

$\alpha = 0,05$

(ii) $z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

(iii)




Reject $H_0$ if $|z| > 1,96$

(iv) $n = 36$; $\bar{X} = 23$; $s = 7$

$$z = \frac{23 - 24}{7/\sqrt{36}} = -0,857$$

(v) Do NOT Reject $H_0$. Mean dress sold per day is NOT significantly different from 24 at $\alpha = 0,05$

10.16) (i) $H_0: \mu \geq 20$

$H_A: \mu < 20$

$\alpha = 0,05$

(ii) $z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

(iii)


Reject $H_0$ if $z < -1,665$

(iv) $n = 100$; $\bar{X} = 18,1$; $s = 5,4$

$$z = \frac{18,1 - 20}{5,4/\sqrt{100}} = -3,52$$

(v) Reject $H_0$. Mean Number of Patients is significantly less than 20 at $\alpha = 0,05$

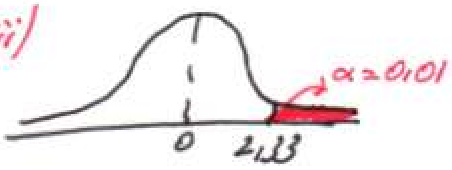
10.17) (i) $H_0: \mu \le 8$

$H_A: \mu > 8$

$\alpha = 0{,}01$

(ii) $z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

(iii)




Reject $H_0$ if $z > 2{,}33$

(iv) $n = 49$; $\bar{X} = 8{,}2$; $\sigma = 0{,}4$

$$z = \frac{8{,}2 - 8}{0{,}4 / \sqrt{49}} = 3{,}5$$

(v) Reject $H_0$. Overfilling is significant at $\alpha = 0{,}05$.

* when we make hypothesis testing for population proportion, we use the test statistics

$$\boxed{z = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}} \quad \text{where} \quad \boxed{\hat{p} = \frac{X}{n}}$$

**11.9** A self-service gasoline station operator claims that at least 50% of the drivers who patronize the station are women. To check this claim, a random sample was taken, and it was found that 85 of 200 patrons were women. Test the null hypothesis $p = 0.50$ against a suitable alternative hypothesis at the 0.01 level of significance.

**11.10** Repeat Exercise 11.9 with the new information that 90 (not 85) of the 200 randomly selected drivers are women. Otherwise, this exercise is the same as Exercise 11.9.

**11.11** In a random sample of 500 cars making a left turn at a certain intersection, 169 pulled into the wrong lane. Test the null hypothesis that the actual proportion of drivers who make this mistake (at the given intersection) is 0.30 against the alternative hypothesis that this figure is too low. Use the 0.01 level of significance.

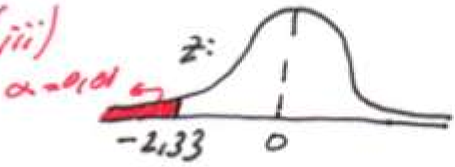
11.9) (i) $H_0: p \geq 0{,}50$

$H_A: p < 0{,}50$

$\alpha = 0{,}01$

(ii) $$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

(iii)




Reject $H_0$ if $Z < -2{,}33$

(iv) $n = 200$; $X = 85$; $\hat{p} = \frac{85}{200} = 0{,}425$

$$Z = \frac{0{,}425 - 0{,}5}{\sqrt{\frac{0{,}5 \cdot 0{,}5}{200}}} = -2{,}12$$

(v) Do NOT Reject $H_0$. ~~Mean~~ Proportion of women is NOT less than 50%. at $\alpha = 0{,}01$

11.10) (iv) $\hat{p} = \frac{90}{200} = 0{,}45$ and $Z = \frac{0{,}45 - 0{,}5}{\sqrt{\frac{0{,}5 \cdot 0{,}5}{200}}} = -1{,}141$

(v) Do NOT Reject $H_0$.

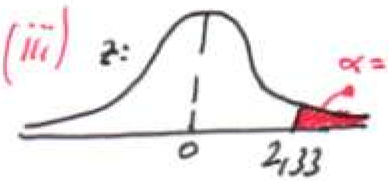
10.11) (i) $H_0: p \leq 0{,}30$

$H_A: p > 0{,}30$

$\alpha = 0{,}01$

(ii) $$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

(iii)




Reject $H_0$ if $Z > 2{,}33$

(iv) $n = 500$; $X = 169$; $\hat{p} = \frac{169}{500} = 0{,}338$

$$Z = \frac{0{,}338 - 0{,}30}{\sqrt{\frac{0{,}30 \cdot 0{,}70}{500}}} = 1{,}854$$

Do NOT Reject $H_0$. Proportion of cars making a left turn is NOT significantly greater than 30%.