MULTIPLE REGRESSION ANALYSIS: INFERENCE

Remember; PRF: $E(y) = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_k X_{ki}$

SRF: $\hat{y} = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \dots + \hat{\beta}_k X_{ki}$

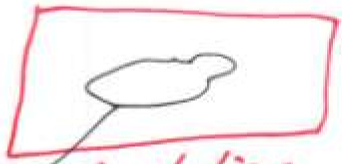
Population Parameters		Sample Statistics
(Unknown Constants)		(Known Variables)

$$E(y) \qquad\qquad\qquad\qquad \hat{y}$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_k \end{bmatrix} \xleftarrow{\text{INFERENCE}} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = \hat{\beta}$$

(i) Hypothesis Testing
(ii) Confidence Interval

$y = \beta X + u \qquad\qquad y = \hat{\beta} X + \hat{u}$



Population

Random Sample of size n

Sample: $\underline{y}$ $\underline{X}$

Example: Consider the model:

$$\log M_{2i} = \beta_1 + \beta_2 \log r_i + \beta_3 \log gdp_i + \beta_4 \log cpi_i + u_i$$

where; M_2 : (Italian) Money Demand
r : rate (Opportunity cost of holding money)
gdp : Gross Domestic Product (Represents Income)
cpi : Consumer Price Index (Represents Prices)

The model is estimated using $n = 92$ sample data:

$$\widehat{\log M_{2i}} = 0{,}63 + 0{,}0741 \log r_i + 0{,}771 \log gdp_i + 0{,}02 \log cpi_i$$

S.E.: (0,28) (0,14) (0,019) (0,18)

For example, β_2 measures the change in the (mean) value of Y ($\log M_2$) caused by a unit increase in X_2 ($\log r$), holding the values of X_3 and X_4 (log gdp and log cpi) constant (ceteris palabus). Namely, if $\log r$ increases by 1 unit, on the average, it will lead $\log M_2$ to increase (Because $\hat{\beta}_2 > 0$) by 0,0741 units, holding other X_j constant.

Remember, in log-log models, $\hat{\beta}_j$ represents the estimated elasticities. So, we may interpret $\hat{\beta}_2$ as: If r increases by 1%, M_2 is expected to increase by 0,0741 %, ceteris palabus.

(I) TESTING the OVERALL (joint) SIGNIFICANCE

Note that, the t-test and F-test are valid under the assumption that residuals term are i.i.d Normally distributed. Remember;

$$u_i \sim NID(0; \sigma^2)$$

(i) Residuals are independent

(ii) Residuals' mean is 0

(iii) Residuals have constant Variance σ^2

(iv) Residuals have Normal distribution

If ALL the model coefficients (except intercept: β_1) are equal to 0, the model is invalid. So, we test the validity of the model as follows:

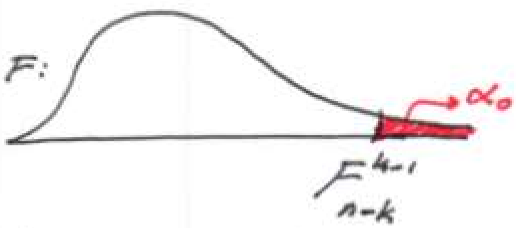
(i) $H_0: \beta_2 = \beta_3 = \cdots = \beta_k = 0$ → The model is INVALID

H_A: At least one β_j is NON-zero for $j = 2, 3, \ldots, k$ → Model somehow works

$\alpha = \alpha_0$

(ii) $$F = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$ → df for num $= k-1$ → df for denom $= n-k$

(iii)



Reject H_0 if $F > F^{k-1}_{n-k}$

(iv) Calculate F by ANOVA table or using R^2 formula

ANOVA Table

Source	df	S.S	M.S	F
Regression	$k-1$	ESS	$m(ESS) = \frac{ESS}{k-1}$	$F = \frac{m(ESS)}{m(RSS)}$
Residual	$n-k$	RSS	$m(RSS) = \frac{RSS}{n-k}$	—
Error	$n-1$	TSS	—	—

(v) Decision: Reject H_0, The model is valid at $\alpha = \alpha_0$

OR

Do NOT Reject H_0, The model is NOT valid at $\alpha = \alpha_0$

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Base Question

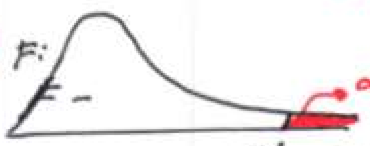
Answer 4 f) (i) $H_0: \beta_2 = \beta_3 = 0$

H_A: At least one β_j is NON-ZERO

$\alpha = 0{,}05$ → Usual α if NOT given

(ii)

$$F = \frac{ESS/2}{RSS/2} \quad \begin{matrix} \to k-1 = 3-1 = 2 \\ \to n-k = 5-3 = 2 \end{matrix}$$

(iii) 

Reject H_0 if $F > 19$

(iv) ANOVA

Source	df	SS	MS	F
Reg.	2	106	$\frac{106}{2} = 53$	$\frac{53}{3} = 17.7$
Res.	2	6	$\frac{6}{2} = 3$	—
TOTAL	4	112	—	—

(v) $F = 17.7 < 19$, do NOT Reject H_0.
The Model is NOT Valid at $\alpha = 0{,}05$

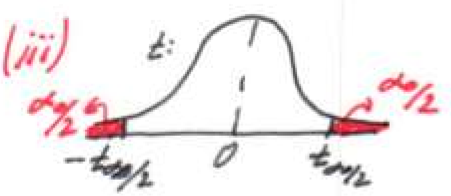
(II) TESTING THE INDIVIDUAL PARAMETERS

The individual Significance (OR comparison with a constant) of Regression Coefficients are tested using a t-test.

(i)

$H_0: \beta_j = c$	$H_0: \beta_j \leq c$	$H_0: \beta_j \geq c$
$H_A: \beta_j \neq c$	$H_A: \beta_j > c$	$H_A: \beta_j < c$
$\alpha = \alpha_0$	$\alpha = \alpha_0$	$\alpha = \alpha_0$

(ii) $$t = \frac{\hat{\beta}_j - c}{SE(\hat{\beta}_j)} \; ; \; df = n-k$$

(iii) 

Reject H_0 if $|t| > t_{\alpha_0/2}$

Reject H_0 if $t > t_{\alpha_0}$

Reject H_0 if $t < -t_{\alpha_0}$

(iv) – (v) Calculate t, decide and Conclude

Base Question

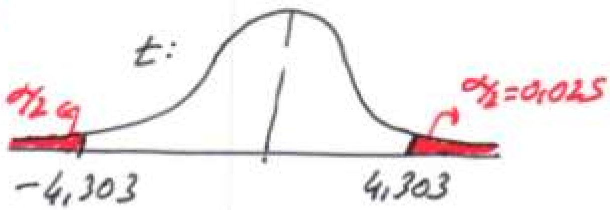
Answers g) (i) $H_0: \beta_2 = 0$

$H_A: \beta_2 \neq 0$

$\alpha = 0{,}05$

(ii) $t = \dfrac{\hat{\beta}_2 - \beta_2}{SE(\hat{\beta}_2)}$; $df = 5 - 3 = 2$

(iii)



Reject H_0 if $|t| > 4{,}303$

(iv) Remember;

$$Var\text{-}Cov(\hat{\beta}) = \begin{array}{c|ccc} & \hat{\beta}_1 & \hat{\beta}_2 & \hat{\beta}_3 \\ \hline \hat{\beta}_1 & 80{,}1 & 6{,}75 & -12 \\ \hat{\beta}_2 & 6{,}75 & 0{,}75 & -1{,}125 \\ \hat{\beta}_3 & -12 & -1{,}125 & 1{,}875 \end{array}$$

$Cov(\hat{\beta}_2, \hat{\beta}_2) = Var(\hat{\beta}_2) = 0{,}75$

$SE(\hat{\beta}_2) = \sqrt{0{,}75} = 0{,}866$

SRF: $\hat{y} = \underbrace{8}_{=\hat{\beta}_1} + \underbrace{2{,}5}_{=\hat{\beta}_2} X_2 - \underbrace{1{,}5}_{=\hat{\beta}_3} X_3$

$t = \dfrac{2{,}5 - 0}{0{,}866} = 2{,}887$

(v) Do NOT Reject H_0. β_2 is NOT significantly different from 0 (or in short, NOT significant) at $\alpha = 0{,}05$.

(III) TESTING LINEAR EQUATIONS of COEFFICIENTS by t-test.

Remember; if V and W are random variables;

$$Var(aV + bW) = a^2 \cdot Var(V) + b^2 \cdot Var(W) + 2ab\, Cov(V, W)$$

(like: $(aV + bW)^2 = a^2 V^2 + b^2 W^2 + 2ab\, VW$)

So, when testing $H_0: a_1\beta_1 + a_2\beta_2 + \dots + a_k\beta_k = 0$, we find S.E. of the test to be used via $Var\text{-}Cov(\hat{\beta})$ matrix:

$$\boxed{Var(a_1\hat{\beta}_1 + a_2\hat{\beta}_2 + \dots + a_k\hat{\beta}_k) = \sum_{j=1}^{k} a_j^2\, Var(\hat{\beta}_j) + 2 \cdot \sum_{i<j} Cov(\hat{\beta}_i, \hat{\beta}_j)}$$

Base Question

Answer h)

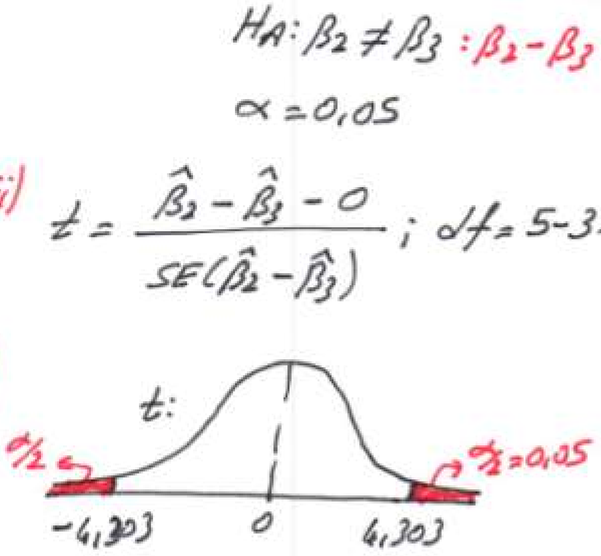
(i) $H_0: \beta_2 = \beta_3 : \beta_2 - \beta_3 = 0$

$H_A: \beta_2 \neq \beta_3 : \beta_2 - \beta_3 \neq 0$

$\alpha = 0,05$

(ii) $t = \dfrac{\hat{\beta}_2 - \hat{\beta}_3 - 0}{SE(\hat{\beta}_2 - \hat{\beta}_3)}$; $df = 5-3 = 2$

(iii)



Reject H_0 if $|t| > 4,303$

(iv)

$$\text{Var-Cov}(\hat{\beta}) = \begin{array}{cc} & \begin{array}{ccc} \hat{\beta}_1 & \hat{\beta}_2 & \hat{\beta}_3 \end{array} \\ \begin{array}{c} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{array} & \begin{bmatrix} 80,1 & 6,75 & -12 \\ 6,75 & 0,75 & -1,125 \\ -12 & -1,125 & 1,875 \end{bmatrix} \end{array}$$

$Var(\hat{\beta}_2 - \hat{\beta}_3) = Var(\hat{\beta}_2) + Var(\hat{\beta}_3) - 2Cov(\hat{\beta}_2, \hat{\beta}_3)$

$= 0,75 + 1,875 - 2.(-1,125)$

$= 4,875$

$SE(\hat{\beta}_2 - \hat{\beta}_3) = \sqrt{4,875} = 2,208$

SRF: $\underbrace{8}_{=\hat{\beta}_1} + \underbrace{2,5}_{=\hat{\beta}_2} X_2 - \underbrace{1,5}_{=\hat{\beta}_3} X_3$

$t = \dfrac{2,5 - (-1,5) - 0}{2,208} = 1,812$

(v) Do NOT Reject H_0. β_2 is NOT significantly different from β_3 at $\alpha = 0,05$.

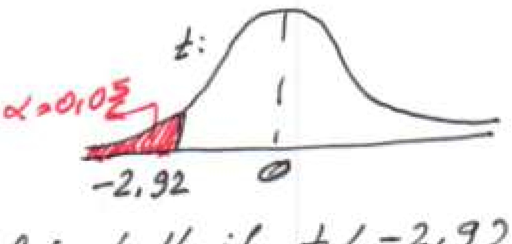
i)

(i) $H_0: \beta_2 + \beta_3 \geq 1,5$

$H_A: \beta_2 + \beta_3 < 1,5$

$\alpha = 0,05$

(ii) $t = \dfrac{\hat{\beta}_2 + \hat{\beta}_3 - 1,5}{SE(\hat{\beta}_2 + \hat{\beta}_3)}$; $df = 2$



Reject H_0 if $t < -2,92$

(iv) $Var(\hat{\beta}_2 + \hat{\beta}_3) = Var(\hat{\beta}_2) + Var(\hat{\beta}_3) + 2Cov(\hat{\beta}_2, \hat{\beta}_3)$

$= 0,75 + 1,875 + 2.(-1,125)$

$= 0,375$

$SE(\hat{\beta}_2 + \hat{\beta}_3) = \sqrt{0,375} = 0,612$

$t = \dfrac{2,5 + (-1,5) - 1,5}{0,612} = -1,634$

(v) Do NOT Reject H_0. $\beta_2 + \beta_3$ is NOT significantly less than 1,5 at $\alpha = 0,05$

(IV) CONFIDENCE INTERVAL for β_j

We use the same formula as we used in two-variable case (only with change $df = n-k$ is NOT necessarily $n-2$)

$(1-\alpha).100\%$ C.I. for β_j is;

$$\hat{\beta}_j \pm t_{\alpha/2;n-k} \cdot SE(\hat{\beta}_j)$$

8.16) In studying the demand for farm tractors in the United States for the periods 1921–1941 and 1948–1957, Griliches[†] obtained the following results.

$$\widehat{\log Y_t} = \text{constant} - \underset{(0.231)}{0.519} \log X_{2t} - \underset{(0.477)}{4.933} \log X_{3t} \qquad R^2 = 0.793$$

where Y_t = value of stock of tractors on farms as of January 1, in 1935–1939 dollars, X_2 = index of prices paid for tractors divided by an Index of prices received for all crops at time $t-1$, X_3 = interest rate prevailing in year $t-1$, and the estimated standard errors are given in the parentheses.

a. Interpret the preceding regression.
b. Are the estimated slope coefficients individually statistically significant? Are they significantly different from unity?
c. Use the analysis of variance technique to test the significance of the overall regression. *Hint:* Use the R^2 variant of the ANOVA technique.
d. How would you compute the interest-rate elasticity of demand for farm tractors?
e. Find 95% Confidence intervals for slope coefficients.

8.16) a) Note that $\hat{\beta}_2$ and $\hat{\beta}_3$ are estimated elasticities. The elasticity of stock with respect to X_2 is $-0{,}519$ and the elasticity of stock with respect to X_3 is $-4{,}933$.

Namely, 1% increase in X_2, on the average, is expected to lead a 0,52% decrease in stock of tracktors and likewise 1% increase in X_3, on the average, it expected to lead a 4,93% decrease in stock of tracktors.

b) (i) $H_0: \beta_2 = 0$ $(\beta_2 = {}^{-}1)$ $\qquad$ $H_0: \beta_3 = 0$ $(\beta_3 = {}^{-}1)$

$H_A: \beta_2 \neq 0$ $(\beta_2 \neq {}^{-}1)$ $\qquad$ $H_A: \beta_3 \neq 0$ $(\beta_3 \neq {}^{-}1)$

$\alpha = 0{,}05$ $\qquad$ $\alpha = 0{,}05$

(ii) $n = (1941 - 1921 + 1) + (1957 - 1948 + 1) = 41$

$$t = \frac{\hat{\beta}_j - \beta_j}{SE(\hat{\beta}_j)} \; ; \; df = n - 3 = 41 - 3 = 38$$

(iii)

Reject H_0 if $|t| > 2{,}021$

(iv) $SE(\hat{\beta}_2) = 0{,}231$

$$t = \frac{-0{,}519 - 0}{0{,}231} = {}^{-}2{,}247 \quad \left(t = \frac{-0{,}519 - (-1)}{0{,}231} = 2{,}082 \right)$$

$SE(\hat{\beta}_3) = 0{,}477$

$$t = \frac{-4{,}933 - 0}{0{,}477} = -10{,}34 \quad \left(t = \frac{-4{,}933 - (-1)}{0{,}477} = -8{,}25 \right)$$

(v) Reject H_0 (Do NOT Reject H_0)

Reject H_0 (Reject H_0)

β_2 is significantly different from 0 but NOT significantly different from 1. Stock of tracktors w.r.t X_2 is inelastic. at $\alpha=0{,}05$

β_3 is significantly different from both 0 and 1. In fact, β_3 is significantly greater than 1. Stock of tracktors w.r.t X_3 is elastic at $\alpha = 0{,}05$.

c) (i) $H_0: \beta_2 = \beta_3 = 0$

H_A: At least one β_j is NON-zero

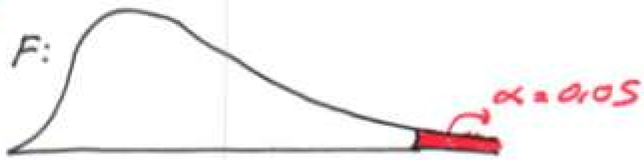
$\alpha = 0,05$

(ii) $$F = \frac{R^2/(3-1)}{(1-R^2)/(41-3)} ;$$

$(3-1) \rightarrow$ df NUM = 2

$(41-3) \rightarrow$ df DENOM = 38

(iii)



$F_{38}^{2} = 3,23$

Reject H_0 if $F > 3,23$

(iv) $$F = \frac{0,793/2}{(1-0,793)/38} = 72,8$$

(v) $72,8 > 3,23$ So

Reject H_0.

The model is valid at $\alpha = 0,05$.

(Note that, p-value = 0,0000 Model is valid at any α)

d) $\hat{\beta}_2$ and $\hat{\beta}_3$ are already estimated elasticities.

e) $t_{\alpha/2} = 2,021$ (part b)

95% C.I. for β_2 is;

$$\hat{\beta}_2 \pm t_{\alpha/2} \cdot SE(\hat{\beta}_2)$$

$$-0,519 \pm 2,021 \cdot (0,231)$$

$$(-0,986 ; -0,052$$

95% C.I. for β_3 is;

$$\hat{\beta}_3 \pm t_{\alpha/2} \cdot SE(\hat{\beta}_3)$$

$$-4,933 \pm 2,021 \cdot (0,477)$$

$$(-5,897 ; -3,969)$$

8.14. From a sample of 209 firms, Wooldridge obtained the following regression results*:

$$\widehat{\log(\text{salary})} = 4.32 + 0.280\log(\text{sales}) + 0.0174\,\text{roe} + 0.00024\,\text{ros}$$
$$\text{se} = (0.32) \quad (0.035) \qquad (0.0041) \qquad (0.00054)$$
$$R^2 = 0.283$$

where salary = salary of CEO
sales = annual firm sales
roe = return on equity in percent
ros = return on firm's stock

and where figures in the parentheses are the estimated standard errors.

a. Interpret the preceding regression taking into account any prior expectations that you may have about the signs of the various coefficients.
b. Which of the coefficients are *individually* statistically significant at the 5 percent level?
c. What is the overall significance of the regression? Which test do you use? And why?
d. Can you interpret the coefficients of roe and ros as elasticity coefficients? Why or why not?

8.14) a) A priori, all coefficients signs are expected to be positive and they are positive. Salary's elasticity w.r.t. sales is 0,280. Namely, 1% increase in sales, on the average, is expected to increase salary by 0,28%. Remember, log-lin model stands for growth rate modeling. We interpret $\hat{\beta}_3$ and $\hat{\beta}_4$ as follows: A unit increase in roe leads to increase salary by 1,74% (we multiply by 100) and a unit increase in ros leads to increase salary by 0,024%. However, these interpretations are valid only if corresponding $\hat{\beta}_j$ are significant.

$x_j \Rightarrow y$

Remember ⇒ log-log Model: % change ⇒ % Change
log-lin Model: 100. Unit Change ⇒ % Change
lin-log Model: $\frac{\%\ \text{Change}}{100}$ ⇒ Unit Change

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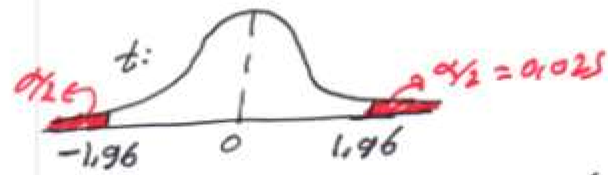
b) (i) $H_0: \beta_2 = 0$ $\quad$ $H_0: \beta_3 = 0$ $\quad$ $H_0: \beta_4 = 0$

$H_A: \beta_2 \neq 0$ $\quad$ $H_A: \beta_3 \neq 0$ $\quad$ $H_A: \beta_4 \neq 0$

$\alpha = 0{,}05$ $\quad$ $\alpha = 0{,}05$ $\quad$ $\alpha = 0{,}05$

(ii) $$t = \frac{\hat{\beta}_j - \beta_j}{SE(\hat{\beta}_j)} \; ; df = n-4 = 209-4 = 205$$

(iii)



Reject H_0 if $|t| > 1{,}96$

(iv) $t = \frac{0{,}280 - 0}{0{,}035} = 8$ $\quad$ $t = \frac{0{,}0174 - 0}{0{,}0041} = 4{,}24$ $\quad$ $t = \frac{0{,}00024 - 0}{0{,}00054} = 0{,}44$

(v) Reject H_0 $\quad$ Reject H_0 $\quad$ Do NOT Reject H_0.

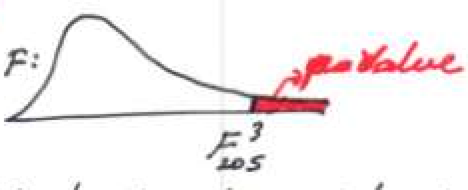
Sales and Roe are significant variables for Salary but Ros is NOT significant at $\alpha = 0{,}05$. Ros should be omited from the model.

c) (i) $H_0: \beta_2 = \beta_3 = \beta_4 = 0$

H_A: At least one β_j is NON-zero

(ii) $$F = \frac{M(ESS) \rightarrow df = k-1 = 3}{M(RSS) \rightarrow df = n-k = 205}$$

(iii)



Reject H_0 if p-Value $< \alpha$

(iv) $$F = \frac{R^2/3}{(1-R^2)/205} = \frac{0{,}283/3}{(1-0{,}283)/205} = 26{,}97$$

p-Value $< 0{,}01$

(v) Regression is Highly significant since p-value $< 0{,}01$

d) No, because log-log Model coefficients give estimated elasticities. Log-lin model coefficients give estimated growth rates.

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8.17. Consider the following wage-determination equation for the British economy* for the period 1950–1969:

$$\hat{W}_t = \underset{(1.129)}{8.582} + \underset{(0.080)}{0.364}(\mathrm{PF})_t + \underset{(0.072)}{0.004}(\mathrm{PF})_{t-1} - \underset{(0.658)}{2.560}U_t$$

$$R^2 = 0.873 \qquad \mathrm{df} = 15$$

where W = wages and salaries per employee
PF = prices of final output at factor cost
U = unemployment in Great Britain as a percentage of the total number of employees of Great Britain
t = time

(The figures in the parentheses are the estimated standard errors.)

a. Interpret the preceding equation.
b. Are the estimated coefficients individually significant?
c. What is the rationale for the introduction of $(\mathrm{PF})_{t-1}$?
d. Should the variable $(\mathrm{PF})_{t-1}$ be dropped from the model? Why?
e. How would you compute the elasticity of wages and salaries per employee with respect to the unemployment rate U?

8.18. A variation of the wage-determination equation given in exercise 8.17 is as follows†:

$$\hat{W}_t = \underset{(0.797)}{1.073} + \underset{(0.812)}{5.288}V_t - \underset{(0.111)}{0.116}X_t + \underset{(0.022)}{0.054}M_t + \underset{(0.019)}{0.046}M_{t-1}$$

$$R^2 = 0.934 \qquad \mathrm{df} = 14$$

where W = wages and salaries per employee
V = unfilled job vacancies in Great Britain as a percentage of the total number of employees in Great Britain
X = gross domestic product per person employed
M = import prices
M_{t-1} = import prices in the previous (or lagged) year

(The estimated standard errors are given in the parentheses.)

a. Interpret the preceding equation.
b. Which of the estimated coefficients are individually statistically significant?
c. What is the rationale for the introduction of the X variable? A priori, is the sign of X expected to be negative?
d. What is the purpose of introducing both M_t and M_{t-1} in the model?
e. Which of the variables may be dropped from the model? Why?
f. Test the overall significance of the observed regression.

g. Compare the models in 8.17 and 8.18. Which model do you prefer?

8.17) a) A unit increase in PF, on the average, is expected to lead to increase wage by 0,364 units. We'll see that previous PF (PF_{t-1}) is NOT significant. A unit increase in U, on the average, is expected to lead to decrease wage by 2,56 units.

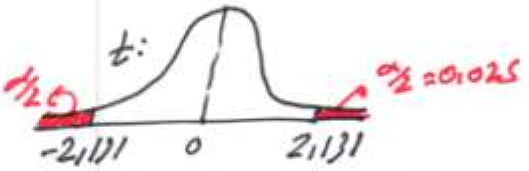
b) (i) $H_0: \beta_2 = 0$ $\quad H_A: \beta_2 \neq 0$ $\quad \alpha = 0,05$

$H_0: \beta_3 = 0$ $\quad H_A: \beta_3 \neq 0$ $\quad \alpha = 0,05$

$H_0: \beta_4 = 0$ $\quad H_A: \beta_4 \neq 0$ $\quad \alpha = 0,05$

(ii) $t = \dfrac{\hat{\beta}_j - \beta_j}{SE(\hat{\beta}_j)}$; $df = 15$ ($df = n - 4$)

(iii)



Reject H_0 if $|t| > 2,131$

(iv) $t = \dfrac{0,364 - 0}{0,080} = 4,55$ $\qquad t = \dfrac{0,004 - 0}{0,072} = 0,06$ $\qquad t = \dfrac{-2,560 - 0}{0,168} = 15,24$

(v) Reject H_0 $\qquad$ Do NOT Reject H_0 $\qquad$ Reject H_0.

PF and U are significant but previous PF is NOT significant at $\alpha = 0,05$

c) Last Year PF is thought to effect Wages.

d) Yes, because it is NOT significant.

e) Remember; $E = \dfrac{dy}{dx} * \dfrac{X}{y}$ is elasticity of y w.r.t X.

We have; $W_t = \beta_1 + \beta_2 (PF)_t + \beta_3 (PF)_{t-1} + \beta_4 U_t$

$$\frac{dW_t}{dU_t} = \beta_4 \qquad E(U) = \frac{dW_t}{dU_t} \cdot \frac{U_t}{W_t} = \beta_4 \cdot \frac{U_t}{W_t} = \hat{\beta}_4 \cdot \frac{U_t}{\hat{W}_t}$$

Given U_t, compute $\hat{W}_t$ with average values of PF_t and PF_{t-1}.

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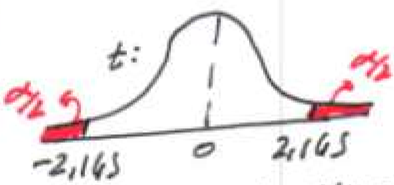
Then, put in the formula of $E(U)$.

8.18) a) A unit increase in V, on the average, is expected to lead to increase wage by 5,288 units. X is NOT significant. A unit increase in M_t (M_{t-1}), on the average, is expected to lead to increase wage by 0,054 (0,046) units.

b)



Reject Ho if $|t| > 2{,}165$

$(df = n-5 = 14)$

V (β_2): $t = \dfrac{5{,}288-0}{0{,}182} = 29{,}05$

X (β_3): $t = \dfrac{-0{,}116-0}{0{,}111} = 1{,}05$

M_t (β_4): $t = \dfrac{0{,}054-0}{0{,}022} = 2{,}45$

M_{t-1} (β_5): $t = \dfrac{0{,}046-0}{0{,}019} = 2{,}42$

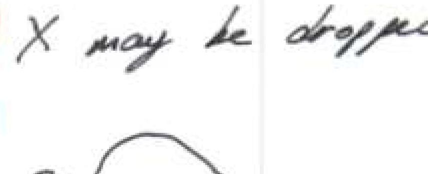
V, M_t and M_{t-1} are significant variables, X is NOT significant.

c) GDP is expected to be related positively with wages however its sign is negative. In fact, it is NOT significant.

d) Previous year's import prices ~~also~~ as well as current import prices is thought to effect wages.

e) X may be dropped from the model since it is NOT significant.

f)



$F^{4}_{14} = 3{,}11$

Reject Ho if $F > 3{,}11$

$$F = \frac{R^2/4}{(1-R^2)/14} = \frac{0{,}934/4}{(1-0{,}934)/14} = 49{,}53$$

Reject Ho. Model is significant at $\alpha = 0{,}05$. (Note that model is significant at any reasonable α value)

g) Model I

$$\bar{R}^2 = 1-(1-R^2)\cdot\frac{n-1}{n-k} = 1-(1-0{,}873)\cdot\frac{19-1}{19-4} = 0{,}8476$$

Model II

$$\bar{R}^2 = 1-(1-0{,}936)\cdot\frac{19-1}{19-5} = 0{,}9151$$

Choose Model II because its $\bar{R}^2$ is higher.

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(V) TESTING LINEAR EQUATIONS of COEFFICIENTS by F-test

The usual multiple regression model is called **Full model** here. We restrict the model by putting linear equations in the full model, which is called **Restricted model**.

We have;

H_0: linear equations (Supports Restricted Model)
H_A: At least one restriction is NOT Valid (Supports Full Model)

The test statistics is:

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R^2_{UR} - R^2_R)/m}{(1-R^2_{UR})/(n-k)}$$

where; m = Number of restrictions in H_0
R: Restricted Model, UR: Unrestricted (Full) Model
RSS: Residual sum of squares
k = Number of coefficients in the full model.

Example 4 $\hat{M}_2 = -0{,}4 - 0{,}075\,GDP + 7{,}6\,CPI - 0{,}68\,r$
$R^2 = 0{,}986$; $n = 92$

$\hat{M}_2 = -1{,}18 + 6{,}48\,CPI$
$R^2 = 0{,}863$

Which model is better? Test at $\alpha = 0{,}05$

Answer 4 Note that, $R^2_U = 0{,}986$ and $R^2_R = 0{,}863$

$\hat{M}_2 = \beta_1 + \beta_2\,GDP + \beta_3\,CPI + \beta_4\,r \rightarrow$ **Full Model**

$\hat{M}_2 = \beta_1 + \beta_3\,CPI \rightarrow$ **Restricted Model**

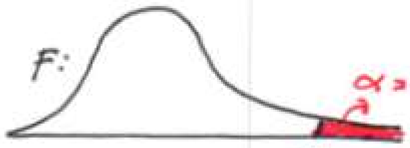
(i) $H_0: \beta_2 = \beta_4 = 0$ ($H_0: \beta_2 = 0$ AND $\beta_4 = 0$)

H_A: At least one Restriction is Not Valid

$\alpha = 0,05$

(ii) $$F = \frac{(R^2_{UR} - R^2_R)/2}{(1 - R^2_{UR})/(92-4)}$$ → df num = 2 → df denom = 88

(iii)



$F^2_{88} = 3,15$

Reject H_0 if $F > 3,15$

(iv) $$F = \frac{(0,986 - 0,863)/2}{(1 - 0,986)/88} = 386,6$$

(v) Reject H_0. Data Supports Full Model.

8.32. Return to exercise 1.7, which gave data on advertising impressions retained and advertising expenditure for a sample of 21 firms. In exercise 5.11 you were asked to plot these data and decide on an appropriate model about the relationship between impressions and advertising expenditure. Letting Y represent impressions retained and X the advertising expenditure, the following regressions were obtained:

Model I: $\hat{Y}_i = 22.163 + 0.3631X_i$

$\quad$ se = (7.089) $\quad$ (0.0971) $\qquad r^2 = 0.424$

Model II: $\hat{Y}_i = 7.059 + 1.0847X_i - 0.0040X_i^2$

$\quad$ se = (9.986) $\quad$ (0.3699) $\quad$ (0.00199) $\qquad R^2 = 0.53$

a. Interpret both models.
b. Which is a better model? Why?
c. Which statistical test(s) would you use to choose between the two models?
d. Are there "diminishing returns" to advertising expenditure, that is, after a certain level of advertising expenditure (the saturation level) it does not pay to advertise? Can you find out what that level of expenditure might be? Show the necessary calculations.

8.32) a) First model assumes a linear model which regresses advertising impressions retained (Y) on advertising expenditure (X) whereas second model assumes a quadratic model.

b) Before making a formal test, we can compare $\bar{R}^2$ values;

$$\bar{R}_1^2 = 1-(1-R_1^2)\cdot\frac{n-1}{n-k} = 1-(1-0{,}424)\cdot\frac{21-1}{21-2} = 0{,}3937$$

$$\bar{R}_2^2 = 1-(1-R_2^2)\cdot\frac{n-1}{n-k} = 1-(1-0{,}53)\cdot\frac{21-1}{21-3} = 0{,}4778$$

Second model seems to be better according to $\bar{R}^2$ comparison. However, hypothesis testing result is more reliable.

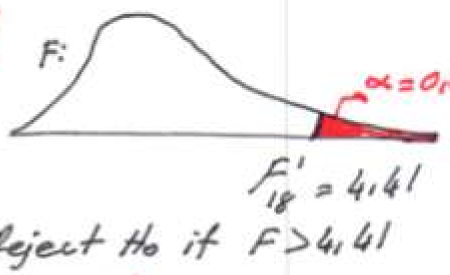
c) We can make either a t-test or an F-test because there's only one restriction. Since we are given both models, I'll apply an F-test.

Full Model: $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{\beta}_3 X_i^2$

$n=21$; $m=1$ restrictions ; $k=3$; $R^2_{UR}=0{,}53$; $R^2_R = 0{,}424$

(i) $H_0: \beta_3 = 0$
$H_A: \beta_3 \neq 0$
$\alpha = 0{,}05$

(ii) $$F = \frac{(R^2_{UR}-R^2_R)/\underbrace{1}_{df_{num}=1}}{(1-R^2_{UR})/\underbrace{(21-3)}_{df_{denom}=18}}$$

(iii) 

Reject Ho if $F > 4{,}41$

(iv) $$F = \frac{(0{,}53-0{,}424)/1}{(1-0{,}53)/18} = 4{,}06$$

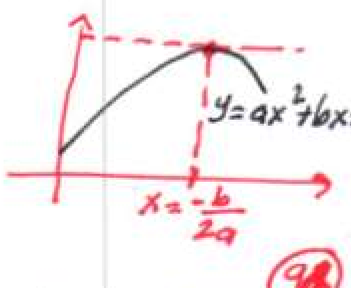
(v) $4{,}06 < 4{,}41$, do NOT Reject Ho. Data Supports Restricted Model. (Linear Model)

d) Assuming Quadratic Model; we have:

$\hat{y}_i = 7{,}059 + 1{,}0847X_i - 0{,}0040X_i^2$

let X_i^* be that level.

$$X_i^* = -\frac{\hat{\beta}_2}{2\cdot\hat{\beta}_3} = -\frac{1{,}0847}{2\cdot(-0{,}0040)} = 135{,}6\,\$$$



8.27. Marc Nerlove has estimated the following cost function for electricity generation*:

$$Y = AX^{\beta} P_1^{\alpha_1} P_2^{\alpha_2} P_3^{\alpha_3} u \qquad (1)$$

where Y = total cost of production
X = output in kilowatt hours
P_1 = price of labor input
P_2 = price of capital input
P_3 = price of fuel
u = disturbance term

Theoretically, the sum of the price elasticities is expected to be unity, i.e., $(\alpha_1 + \alpha_2 + \alpha_3) = 1$. By imposing this restriction, the preceding cost function can be written as $(\alpha_3 = 1 - \alpha_1 - \alpha_2)$

$$(Y/P_3) = AX^{\beta}(P_1/P_3)^{\alpha_1}(P_2/P_3)^{\alpha_2}u \qquad (2)$$

In other words, (1) is an unrestricted and (2) is the restricted cost function.

On the basis of a sample of 29 medium-sized firms, and after logarithmic transformation, Nerlove obtained the following regression results

$$\begin{aligned}\widehat{\ln Y_i} &= -4.93 \quad + 0.94 \ln X_i + 0.31 \ln P_1 \\ \text{se} &= (1.96) \qquad (0.11) \qquad (0.23) \\ &\quad -0.26 \ln P_2 + 0.44 \ln P_3 \\ &\quad (0.29) \qquad (0.07) \qquad \text{RSS} = 0.336\end{aligned} \qquad (3)$$

$$\begin{aligned}\widehat{\ln (Y/P_3)} &= -6.55 + 0.91 \ln X + 0.51 \ln (P_1/P_3) + 0.09 \ln (P_2/P_3) \\ \text{se} &= (0.16) \quad (0.11) \qquad (0.19) \qquad\qquad (0.16) \qquad \text{RSS} = 0.364\end{aligned} \qquad (4)$$

a. Interpret Eqs. (3) and (4).
b. How would you find out if the restriction $(\alpha_1 + \alpha_2 + \alpha_3) = 1$ is valid?

8.27) a) (4) is the estimated function of restricted model where restriction is $H_0: \alpha_1 + \alpha_2 + \alpha_3 = 1$

Since the model is in log-log form, coefficients in (3) are elasticities. For example, elasticity of Total Cost of Production (y) w.r.t price of capital input (P_2) is $\hat{\alpha}_2 = -0{,}26$. Namely, 1% increase in P_2, on the average, is expected to decrease y by 0,26%. Other interpretations are likewise. (+ ⇒ increase)

b) Full Model: $E(\ln Y) = \underbrace{C}_{=\ln A} + \beta \ln X + \alpha_1 \ln P_1 + \alpha_2 \ln P_2 + \alpha_3 \ln P_3$

Restriction: $\alpha_1 + \alpha_2 + \alpha_3 = 1 \Rightarrow \alpha_3 = 1 - \alpha_1 - \alpha_2$

Restricted Model: $E(\ln Y) = C + \beta \ln X + \alpha_1 \ln P_1 + \alpha_2 \ln P_2 + (1 - \alpha_1 - \alpha_2) \ln P_3$

$$E(\ln Y) = C + \beta \ln X + \alpha_1 (\ln P_1 - \ln P_3) + \alpha_2 (\ln P_2 - \ln P_3) + \ln P_3$$

$$E(\ln Y) = \underbrace{D}_{= C + \ln P_3} + \beta \ln X + \alpha_1 \ln (P_1/P_3) + \alpha_2 \ln (P_2/P_3)$$

We have; $n = 29$; $m = 1$; $k = 5$; $RSS_{U} = 0{,}336$; $RSS_R = 0{,}364$

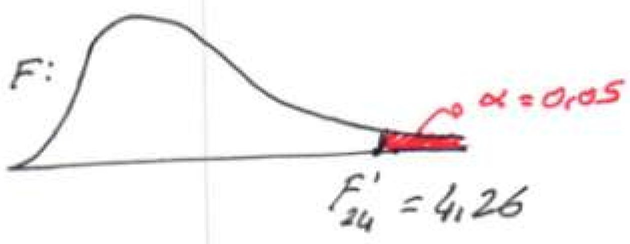
(i) $H_0: \alpha_1 + \alpha_2 + \alpha_3 = 1$ (Restricted)

$H_A: \alpha_1 + \alpha_2 + \alpha_3 \neq 1$ (Full)

$\alpha = 0{,}05$

(ii) $$F = \frac{(RSS_R - RSS_{UR})/1}{RSS_{UR}/(29-5)}$$ → dfNum = 1 → dfDenom = 24

(iii)



(iv) $$F = \frac{0{,}364 - 0{,}336}{0{,}336/24} = 2$$

(v) Do NOT Reject Ho. We may assume unity of elasticity totals.