# OPTIMIZATION Lecture Notes | CHAPTER 3-II

## MODELING EXAMPLES

### * Diet Problem

**2**‡ U.S. Labs manufactures mechanical heart valves from the heart valves of pigs. Different heart operations require valves of different sizes. U.S. Labs purchases pig valves from three different suppliers. The cost and size mix of the valves purchased from each supplier are given in Table 3. Each month, U.S. Labs places one order with each supplier. At least 500 large, 300 medium, and 300 small valves must be purchased each month. Because of limited availability of pig valves, at most 700 valves per month can be purchased from each supplier. Formulate an LP that can be used to minimize the cost of acquiring the needed valves.

| Supplier | Cost Per Value ($) | Percent Large | Percent Medium | Percent Small |
|---|---|---|---|---|
| 1 | 5 | 40 | 40 | 20 |
| 2 | 4 | 30 | 35 | 35 |
| 3 | 3 | 20 | 20 | 60 |
| Required: | | 500 | 300 | 300 |

2) $x_j$: Pig values produced from Supplier $j = 1, 2, 3$

Minimize $z = 5x_1 + 4x_2 + 3x_3$

subject to

$40x_1 + 30x_2 + 20x_3 \geq 500$

$40x_1 + 35x_2 + 20x_3 \geq 300$

$20x_1 + 35x_2 + 60x_3 \geq 300$

$x_1 \leq 700$

$x_2 \leq 700$

$x_3 \leq 700$

$x_1, x_2, x_3 \geq 0$

**1** There are three factories on the Momiss River (1, 2, and 3). Each emits two types of pollutants (1 and 2) into the river. If the waste from each factory is processed, the pollution in the river can be reduced. It costs $15 to process a ton of factory 1 waste, and each ton processed reduces the amount of pollutant 1 by 0.10 ton and the amount of pollutant 2 by 0.45 ton. It costs $10 to process a ton of factory 2 waste, and each ton processed will reduce the amount of pollutant 1 by 0.20 ton and the amount of pollutant 2 by 0.25 ton. It costs $20 to process a ton of factory 3 waste, and each ton processed will reduce the amount of pollutant 1 by 0.40 ton and the amount of pollutant 2 by 0.30 ton. The state wants to reduce the amount of pollutant 1 in the river by at least 30 tons and the amount of pollutant 2 in the river by at least 40 tons. Formulate an LP that will minimize the cost of reducing pollution by the desired amounts. Do you think that the LP assumptions (Proportionality, Additivity, Divisibility, and Certainty) are reasonable for this problem?

1) 3 factories, 2 pollutants

| Factory | Pol. Reduction (per Ton) | Cost |
|---|---|---|
| 1 | 1. 0,10<br>2. 0,45 | 15$ |
| 2 | 1. 0,20<br>2. 0,25 | 10$ |
| 3 | 1. 0,40<br>2. 0,30 | 20$ |

State Requirements.

Pol. 1 → At least 30 tons

Pol. 2 → At least 40 tons.

$x_j$: Reduction of Factory $j$ in tons.

Minimize $z = 15x_1 + 10x_2 + 20x_3$

subject to $0{,}10x_1 + 0{,}20x_2 + 0{,}40x_3 \geq 30$

$0{,}45x_1 + 0{,}25x_2 + 0{,}30x_3 \geq 40$

$x_1, x_2 \geq 0$

* Work Scheduling Problem

**2** During each 4-hour period, the Smalltown police force requires the following number of on-duty police officers: 12 midnight to 4 A.M.—8; 4 to 8 A.M.—7; 8 A.M. to 12 noon—6; 12 noon to 4 P.M.—6; 4 to 8 P.M.—5; 8 P.M. to 12 midnight—4. Each police officer works two consecutive 4-hour shifts. Formulate an LP that can be used to minimize the number of police officers needed to meet Smalltown's daily requirements.

2)

| Period | Required Officers |
|---|---|
| 1- 12AM-4AM | 8 |
| 2- 4AM-8AM | 7 |
| 3- 8AM-12PM | 6 |
| 4- 12PM-4PM | 6 |
| 5- 4PM-8PM | 5 |
| 6- 8PM-12AM | 4 |

$x_j$: # of officers "working" at period $j = 1,2,\ldots,6$

Each officer work for two consequitive periods.

Minimize $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

subject to

$x_6 + x_1 \geq 8$

$x_1 + x_2 \geq 7$

$x_2 + x_3 \geq 6$

$x_3 + x_4 \geq 6$

$x_4 + x_5 \geq 5$

$x_5 + x_6 \geq 4$

$x_j \geq 0 \quad j = 1,2,\ldots,6$

**5** Each day, workers at the Gotham City Police Department work two 6-hour shifts chosen from 12 A.M. to 6 A.M., 6 A.M. to 12 P.M., 12 P.M. to 6 P.M., and 6 P.M. to 12 A.M. The following number of workers are needed during each shift: 12 A.M. to 6 A.M.—15 workers; 6 A.M. to 12 P.M.—5 workers; 12 P.M. to 6 P.M.—12 workers; 6 P.M. to 12 A.M.—6 workers. Workers whose two shifts are consecutive are paid \$12 per hour; workers whose shifts are not consecutive are paid \$18 per hour. Formulate an LP that can be used to minimize the cost of meeting the daily workforce demands of the Gotham City Police Department.

5)

| Period | Required |
|---|---|
| 1) 12AM-6AM | 15 |
| 2) 6AM-12PM | 5 |
| 3) 12PM-6PM | 12 |
| 4) 6PM-12AM | 6 |

Two Consequitive period ⇒ 12 \$/hour

Two NOT Consequitive period ⇒ 18 \$/hour

$X_{ij}$: # of Policers work at periods $i$ and $j$; $i<j$

Consequitive periods workers: $X_{12}, X_{23}, X_{34}, X_{14}$

NON Consequitive periods workers: $X_{13}, X_{24}$

Minimize $z = 12.12(X_{12}+X_{23}+X_{34}+X_{14}) + 12.18(X_{13}+X_{24})$

subject to;

$X_{12}+X_{13}+X_{14} \geq 15$

$X_{12}+X_{23}+X_{24} \geq 5$

$X_{13}+X_{23}+X_{34} \geq 12$

$X_{14}+X_{24}+X_{34} \geq 6$

$X_{ij} \geq 0$ for $i,j \in \{1,2,3,4\}$ and $i<j$

7 Each hour from 10 A.M. to 7 P.M., Bank One receives checks and must process them. Its goal is to process all the checks the same day they are received. The bank has 13 check-processing machines, each of which can process up to 500 checks per hour. It takes one worker to operate each machine. Bank One hires both full-time and part-time workers. Full-time workers work 10 A.M.–6 P.M., 11 A.M.–7 P.M., or Noon–8 P.M. and are paid \$160 per day. Part-time workers work either 2 P.M.–7 P.M. or 3 P.M.–8 P.M. and are paid \$75 per day. The number of checks received each hour is given in Table 6. In the interest of maintaining continuity, Bank One believes it must have at least three full-time workers under contract. Develop a cost-minimizing work schedule that processes all checks by 8 P.M.

TABLE 6

| Time | Checks Received | Number of Machines Required |
|---|---|---|
| 10 A.M. | 5,000 | 10 |
| 11 A.M. | 4,000 | 8 |
| 0 Noon | 3,000 | 6 |
| 1 P.M. | 4,000 | 8 |
| 2 P.M. | 2,500 | 5 |
| 3 P.M. | 3,000 | 6 |
| 4 P.M. | 4,000 | 8 |
| 5 P.M. | 4,500 | 9 |
| 6 P.M. | 3,500 | 7 |
| 7 P.M. | 3,000 | 6 |

7) Full time w. $\underset{6}{X_{10F}}, \underset{7}{X_{11F}}, \underset{8}{X_{0F}} \rightarrow 160\$/day$

Part time w. $\underset{7}{X_{2P}} \cdot \underset{8}{X_{3P}} \rightarrow 75\$/day$

Minimize $z = 160.(X_{10F}+X_{11F}+X_{0F}) + 75.(X_{2P}+X_{3P})$

subject to

$X_{10F} \geq 10$

$X_{10F}+X_{11F} \geq 8$

$X_{10F}+X_{11F}+X_{0F} \geq 6$

$X_{10F}+X_{11F}+X_{0F} \geq 8$

$X_{10F}+X_{11F}+X_{0F}+X_{2P} \geq 5$

$X_{10F}+X_{11F}+X_{0F}+X_{2P}+X_{3P} \geq 9$

$X_{11F}+X_{0F}+X_{2P}+X_{3P} \geq 7$

$X_{0F}+X_{3P} \geq 6$

$X_{ij} \geq 0 \quad X_{ke} \geq 0$

# * Capital Budgeting Problem

**4** A company has nine projects under consideration. The NPV added by each project and the capital required by each project during the next two years is given in Table 9. All figures are in millions. For example, Project 1 will add \$14 million in NPV and require expenditures of \$12 million during year 1 and \$3 million during year 2. Fifty million is available for projects during year 1 and \$20 million is available during year 2. Assuming we may undertake a fraction of each project, how can we maximize NPV?

**TABLE 9**

| | Project | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Year 1 Outflow | 12 | 54 | 6 | 6 | 30 | 6 | 48 | 36 | 18 |
| Year 2 Outflow | 3 | 7 | 6 | 2 | 35 | 6 | 4 | 3 | 3 |
| NPV | 14 | 17 | 17 | 15 | 40 | 12 | 14 | 10 | 12 |

4) $x_j$: Fraction of project $j$ undertaken.

Maximize $14x_1+17x_2+17x_3+15x_4+40x_5+12x_6+14x_7+10x_8+12x_9$

subject to

$$12x_1+54x_2+6x_3+6x_4+30x_5+6x_6+48x_7+36x_8+18x_9 \le 50$$

$$3x_1+7x_2+6x_3+2x_4+35x_5+6x_6+4x_7+3x_8+3x_9 \le 20$$

$$x_1 \le 1 \quad x_2 \le 1 \quad \ldots \quad x_9 \le 1$$

$$x_j \ge 0 \quad \text{for } j=1,2,\ldots,9$$

**5**[†] Finco must determine how much investment and debt to undertake during the next year. Each dollar invested reduces the NPV of the company by 10¢, and each dollar of debt increases the NPV by 50¢ (due to deductibility of interest payments). Finco can invest at most \$1 million during the coming year. Debt can be at most 40% of investment. Finco now has \$800,000 in cash available. All investment must be paid for from current cash or borrowed money. Set up an LP whose solution will tell Finco how to maximize its NPV. Then graphically solve the LP.

5) Each dollar used from cash → -10¢

Each dollar borrowed → +50¢

Invest at most: 1 000 000 \$

Available Cash → 800 000

Dept Used ≤ 0,40. Investment

$x_1$: investment paid from the Cash (in 10 000 \$)

$x_2$: investment paid by dept (in 10 000 \$)

Investment $= x_1 + x_2$ Dept Used ≤ 0,40. Investment

$$x_2 \le 0{,}40\,(x_1+x_2)$$

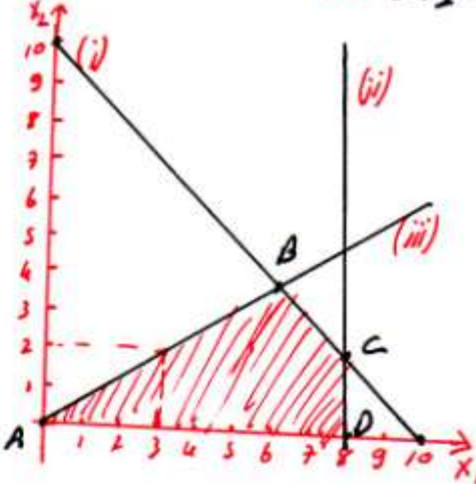
$$3x_2 - 2x_1 \le 0$$

Maximize $z = 0{,}5x_2 - 0{,}1x_1$

subject to (i) $x_1 + x_2 \leq 100$

(ii) $x_1 \leq 80$

(iii) $3x_2 - 2x_1 \leq 0$




B: $2x_1 + 2x_2 = 200$

$\underline{3x_2 - 2x_1 = 0}$

$5x_2 = 200$

$x_2 = 40 \, ; \, x_1 = 60$

C: $x_1 = 80 \, ; \, x_2 = 20$

$z = -0{,}1x_1 + 0{,}5x_2$

A(0;0) $z = 0$

B(60;40) $z = -0{,}1 \cdot 60 + 0{,}5 \cdot 40 = \boxed{14}$ $z_{max}$

C(80;20) $z = -0{,}1 \cdot 80 + 0{,}5 \cdot 20 = 2$

D(80;0) $z = -0{,}1 \cdot 80 = -8$

Max. profit is 140 000 when 600 000 of cash is used and 400 000 borrowed

## * Blending Problem

**1** You have decided to enter the candy business. You are considering producing two types of candies: Slugger Candy and Easy Out Candy, both of which consist solely of sugar, nuts, and chocolate. At present, you have in stock 100 oz of sugar, 20 oz of nuts, and 30 oz of chocolate. The mixture used to make Easy Out Candy must contain at least 20% nuts. The mixture used to make Slugger Candy must contain at least 10% nuts and 10% chocolate. Each ounce of Easy Out Candy can be sold for 25¢, and each ounce of Slugger Candy for 20¢. Formulate an LP that will enable you to maximize your revenue from candy sales.

1)

| | 1. Sugar Candy | 2. Easy Out Candy | Stock |
|---|---|---|---|
| 1. Sugar | $x_{11}$ | $x_{12}$ | 100 |
| 2. Nuts | $x_{21}$ | $x_{22}$ | 20 |
| 3. Chocolade | $x_{31}$ | $x_{32}$ | 30 |
| REQUIREMENTS | ≥10% Nuts<br>≥10% Chocolade | ≥20% Nuts | |
| Sale Price | 20¢ | 25¢ | |

$x_{ij}$: Amount of ingredient $i$ used in product (candy) $j$

Maximize $z = 20\cdot(x_{11} + x_{21} + x_{31}) + 25\cdot(x_{12} + x_{22} + x_{32})$

subject to

$x_{11} + x_{12} \le 100$ $\quad \dfrac{x_{21}}{x_{11} + x_{21} + x_{31}} \ge 0{,}10$

$x_{21} + x_{22} \le 20$

$x_{31} + x_{32} \le 30$ $\quad \dfrac{x_{31}}{x_{11} + x_{21} + x_{31}} \ge 0{,}10$

$x_{ij} \ge 0 \quad i=1,2 \; j=1,2,3$ $\quad \dfrac{x_{22}}{x_{21} + x_{22} + x_{32}} \ge 0{,}20$

**5** Chandler Oil Company has 5,000 barrels of oil 1 and 10,000 barrels of oil 2. The company sells two products: gasoline and heating oil. Both products are produced by combining oil 1 and oil 2. The quality level of each oil is as follows: oil 1—10; oil 2—5. Gasoline must have an average quality level of at least 8, and heating oil at least 6. Demand for each product must be created by advertising. Each dollar spent advertising gasoline creates 5 barrels of demand and each spent on heating oil creates 10 barrels of demand. Gasoline is sold for $25 per barrel, heating oil for $20. Formulate an LP to help Chandler maximize profit. Assume that no oil of either type can be purchased.

5)

| | 1. Gasoline | 2. Heating Oil | Quality level | Available |
|---|---|---|---|---|
| 1. Oil 1 | $x_{11}$ | $x_{12}$ | 10 | 5000 |
| 2. Oil 2 | $x_{21}$ | $x_{22}$ | 5 | 10000 |
| REQ. Qual. | $\ge 8$ | $\ge 6$ | | |
| Adv. | 5/1\$ adv | 10/1\$ adv. | | |
| Sale | 25\$ | 20\$ | | |

$x_{ij}$: Oil $i$ used in Product $j$ and $a_j$: Adv. given for product $j$

Maximize $z = 25(x_{11} + x_{21}) + 20(x_{12} + x_{22})$

subject to

$x_{11} + x_{12} \le 5000$ $\quad \dfrac{10x_{11} + 5x_{21}}{x_{11} + x_{21}} \ge 8$

$x_{21} + x_{22} \le 10000$

$x_{11} + x_{21} = 5a_1$ $\quad \dfrac{10x_{12} + 5x_{22}}{x_{12} + x_{22}} \ge 6$

$x_{12} + x_{22} = 10a_2$ $\quad x_{ij} \ge 0$ for $i = 1,2$ ; $j = 1,2$

**8** Highland's TV-Radio Store must determine how many TVs and radios to keep in stock. A TV requires 10 sq ft of floorspace, whereas a radio requires 4 sq ft; 200 sq ft of floorspace is available. A TV will earn Highland $60 in profits, and a radio will earn $20. The store stocks only TVs and radios. Marketing requirements dictate that at least 60% of all appliances in stock be radios. Finally, a TV ties up $200 in capital, and a radio, $50. Highland wants to have at most $3,000 worth of capital tied up at any time. Formulate an LP that can be used to maximize Highland's profit.

8) $x_1$: # of TV ; $x_2$: # of Radio

| | F.Space | Capital | PROFIT |
|---|---|---|---|
| TV | 10 | 200 | 60 |
| Radio | 4 | 50 | 20 |
| Available | 200 | 3000 | |

$\geq 60\%$ space allocated to radio

Maximize $z = 60x_1 + 20x_2$

subject to $10x_1 + 4x_2 \leq 200$

$200x_1 + 50x_2 \leq 3000$

$$\frac{10x_1}{10x_1 + 4x_2} \geq 0{,}60$$

$x_1, x_2 \geq 0$

**10** Coalco produces coal at three mines and ships it to four customers. The cost per ton of producing coal, the ash and sulfur content (per ton) of the coal, and the production capacity (in tons) for each mine are given in Table 16. The number of tons of coal demanded by each customer are given in Table 17.

The cost (in dollars) of shipping a ton of coal from a mine to each customer is given in Table 18. It is required that the total amount of coal shipped contain at most 5% ash and at most 4% sulfur. Formulate an LP that minimizes the cost of meeting customer demands.

**TABLE 16**

| Mine | Production Cost ($) | Capacity | Ash Content (Tons) | Sulfur Content (Tons) |
|---|---|---|---|---|
| 1 | 50 | 120 | .08 | .05 |
| 2 | 55 | 100 | .06 | .04 |
| 3 | 62 | 140 | .04 | .03 |

**TABLE 17**

| Customer 1 | Customer 2 | Customer 3 | Customer 4 |
|---|---|---|---|
| 80 | 70 | 60 | 40 |

**TABLE 18**

| Mine | Customer 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 4 | 6 | 8 | 12 |
| 2 | 9 | 6 | 7 | 11 |
| 3 | 8 | 12 | 3 | 5 |

| Mine \ Customer | 1 | 2 | 3 | 4 | Cost | Capacity | Ash Content | Sulfur content |
|---|---|---|---|---|---|---|---|---|
| 1 | $x_{11}$ (4) | $x_{12}$ (6) | $x_{13}$ (8) | $x_{14}$ (12) | 50 | 120 | 8% | 5% |
| 2 | $x_{21}$ (9) | $x_{22}$ (6) | $x_{23}$ (7) | $x_{24}$ (11) | 55 | 100 | 6% | 4% |
| 3 | $x_{31}$ (8) | $x_{32}$ (12) | $x_{33}$ (3) | $x_{34}$ (5) | 62 | 140 | 4% | 3% |
| DEMAND | 80 | 70 | 60 | 40 | | | | |

COAL Amount $\Rightarrow$ ash : $\leq 5\%$

sulfur : $\leq 4\%$

$x_{ij}$ : Amount of Coal produced in Mine $i$ and sent to Customer $j$

Minimize $z = 50.(x_{11}+x_{12}+x_{13}+x_{14}) + 55.(x_{21}+x_{22}+x_{23}+x_{24})$
$+ 62.(x_{31}+x_{32}+x_{33}+x_{34}) + 4x_{11} + 6x_{12} + 8x_{13} + 12x_{14}$
$+ 9x_{21} + 6x_{22} + 7x_{23} + 11x_{24} + 8x_{31} + 12x_{32} + 3x_{33} + 5x_{34}$

Subject to

$x_{11}+x_{12}+x_{13}+x_{14} \leq 120$

$x_{21}+x_{22}+x_{23}+x_{24} \leq 100$

$x_{31}+x_{32}+x_{33}+x_{34} \leq 140$

$$\frac{8.(x_{11}+x_{12}+x_{13}+x_{14}) + 6.(x_{21}+x_{22}+x_{23}+x_{24}) + 4.(x_{31}+x_{32}+x_{33}+x_{34})}{\sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij}} \leq 5$$

$$\frac{5.(x_{11}+x_{12}+x_{13}+x_{14}) + 4.(x_{21}+x_{22}+x_{23}+x_{24}) + 3.(x_{31}+x_{32}+x_{33}+x_{34})}{\sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij}} \leq 4$$

$x_{ij} \geq 0 \quad i = 1,2,3; \quad j = 1,2,3,4$

**14** Oilco produces two products: regular and premium gasoline. Each product contains .15 gram of lead per liter. The two products are produced from six inputs: reformate, fluid catalytic cracker gasoline (FCG), isomerate (ISO), polymer (POL), MTBE (MTB), and butane (BUT). Each input has four attributes:

**Attribute 1** Research octane number (RON)
**Attribute 2** RVP
**Attribute 3** ASTM volatility at 70°C
**Attribute 4** ASTM volatility at 130°C

The attributes and daily availability (in liters) of each input are given in Table 21.

The requirements for each output are given in Table 22. The daily demand (in thousands of liters) for each product must be met, but more can be produced if desired. The RON and ASTM requirements are minimums. Regular gasoline sells for 29.49¢/liter, premium gasoline for 31.43¢. Before being ready for sale, 0.15 gram/liter of lead must be removed from each product. Cost of removing 0.10 gr/lt is 8.5¢. At most 38% of each type of gasoline can consist of FCG. Formulate an LP to max. profit.

**TABLE 21**

| | Availability | RON | RVP | ASTM(70) | ASTM(130) |
|---|---|---|---|---|---|
| Reformate | 15,572 | 98.9 | 7.66 | −5 | 46 |
| FCG | 15,434 | 93.2 | 9.78 | 57 | 103 |
| ISO | 6,709 | 86.1 | 29.52 | 107 | 100 |
| POL | 1,190 | 97 | 14.51 | 7 | 73 |
| MTB | 748 | 117 | 13.45 | 98 | 100 |
| BUT | Unlimited | 98 | 166.99 | 130 | 100 |

**TABLE 22**

| | Demand | RON | RVP | ASTM(70) | ASTM(130) |
|---|---|---|---|---|---|
| Regular | 9.8 | 90 | 21.18 | 10 | 50 |
| Premium | 30 | 96 | 21.18 | 10 | 50 |

| | 1. Regular | 2. Premium | Availability | RON | RVP | ASTM70 | ASTM130 |
|---|---|---|---|---|---|---|---|
| 1. Reformate | $x_{11}$ | $x_{12}$ | 15572 | 98,9 | 7,66 | -5 | 46 |
| 2. FCG | $x_{21}$ | $x_{22}$ | 15434 | 93,2 | 9,78 | 57 | 103 |
| 3. ISO | $x_{31}$ | $x_{32}$ | 6709 | 86,1 | 29,52 | 107 | 100 |
| 4. POL | $x_{41}$ | $x_{42}$ | 1190 | 97 | 14,51 | 7 | 73 |
| 5. MTB | $x_{51}$ | $x_{52}$ | 748 | 117 | 13,45 | 98 | 100 |
| 6. BUT | $x_{61}$ | $x_{62}$ | Unlimited | 98 | 166,99 | 130 | 100 |

| | | |
|---|---|---|
| Demand | 9,8 | 30 |
| Profit | 29,49 | 31,43 |
| RON | 90 | 96 |
| RVP | 21,18 | 21,18 |
| ASTM70 | 10 | 10 |
| ASTM130 | 50 | 50 |

* max. FCG ⇒ 38%

$x_{ij}$: Ingredient $i$ used for product $j$

$$\text{Maximize} \quad z = 29{,}49\left(\sum_{i=1}^{6} x_{i1}\right) + 31{,}43\left(\sum_{i=1}^{6} x_{i2}\right) - 8{,}5 \cdot \frac{0{,}15}{0{,}10} \cdot \left(\sum_{i=1}^{6}\sum_{j=1}^{2} x_{ij}\right)$$

subject to

$$x_{11} + x_{12} \le 15572$$

$$x_{21} + x_{22} \le 15434$$

$$x_{31} + x_{32} \le 6709$$

$$x_{41} + x_{42} \le 1190$$

$$x_{51} + x_{52} \le 748$$

$$\frac{98{,}9x_{11} + 93{,}2x_{21} + 86{,}1x_{31} + 97 \cdot x_{41} + 117 \cdot x_{51} + 98x_{61}}{\sum_{i=1}^{6} x_{i1}} \ge 90$$

$$\frac{98{,}9x_{12} + 93{,}2x_{22} + 86{,}1x_{32} + 97x_{42} + 117x_{52} + 98x_{62}}{\sum_{i=1}^{6} x_{i2}} \ge 96$$

$$\frac{7{,}66x_{11} + 9{,}78x_{21} + 29{,}52x_{31} + 16{,}51x_{41} + 13{,}65x_{51} + 166{,}99x_{61}}{\sum_{i=1}^{6} x_{i1}} \geq 21{,}18$$

$$\frac{7{,}66x_{12} + 9{,}78x_{22} + 29{,}52x_{32} + 16{,}51x_{42} + 13{,}65x_{52} + 166{,}99x_{62}}{\sum_{i=1}^{6} x_{i2}} \geq 21{,}18$$

$$\frac{-5.x_{11} + 57x_{21} + 107x_{31} + 7x_{41} + 98x_{51} + 130x_{61}}{\sum_{i=1}^{6} x_{i1}} \geq 10$$

$$\frac{-5.x_{12} + 57x_{22} + 107x_{32} + 7x_{42} + 98x_{52} + 130x_{62}}{\sum_{i=1}^{6} x_{i2}} \geq 10$$

$$\frac{46x_{11} + 103x_{21} + 100x_{31} + 73x_{41} + 100x_{51} + 100x_{61}}{\sum_{i=1}^{6} x_{i1}} \geq 50$$

$$\frac{46x_{12} + 103x_{22} + 100x_{32} + 73x_{42} + 100x_{52} + 100x_{62}}{\sum_{i=1}^{6} x_{i2}} \geq 50$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} \geq 9{,}8$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} \geq 30$$

$$\frac{x_{21}}{\sum_{i=1}^{6} x_{i1}} \leq 0{,}38 \qquad \frac{x_{22}}{\sum_{i=1}^{6} x_{i2}} \leq 0{,}38 \qquad x_{ij} \geq 0 \quad i=1,2 \quad j=1,2,\ldots,6$$

* Production Process Models

2 Furnco manufactures tables and chairs. A table requires 40 board ft of wood, and a chair requires 30 board ft of wood. Wood may be purchased at a cost of $1 per board ft, and 40,000 board ft of wood are available for purchase. It takes 2 hours of skilled labor to manufacture an unfinished table or an unfinished chair. Three more hours of skilled labor will turn an unfinished table into a finished table, and 2 more hours of skilled labor will turn an unfinished chair into a finished chair. A total of 6,000 hours of skilled labor are available (and have already been paid for). All furniture produced can be sold at the following unit prices: unfinished table, $70; finished table, $140; unfinished chair, $60; finished chair, $110. Formulate an LP that will maximize the contribution to profit from manufacturing tables and chairs.

2) Tables & Chairs

$x_1$: # of Unfinished Tables Sold

$x_2$: # of Finished Tables Sold

$y_1$: # of Unfinished Chairs Sold

$y_2$: # of Finished Chairs Sold

$w$: Wood used

| | w: Wood | Labor | PROFIT |
|---|---|---|---|
| ($x_1$) Unfinished Table | 40 | 2 | 70\$ |
| ($x_2$) Finished Table | | 2+3=5 | 140 \$ |
| ($y_1$) Unfinished Chair | 30 | 2 | 60 \$ |
| ($y_2$) Finished Chair | | 2+2=4 | 110 \$ |
| AVAILABLE | 40000 | 6000 | |
| COST | 1 \$ | — | |

Maximize $z = 70x_1 + 140x_2 + 60y_1 + 110y_2 - w$

Subject to $w \leq 40000$

$$2x_1 + 5x_2 + 2x_3 + 4x_4 \leq 6000$$

$$40(x_1 + x_2) + 30(x_3 + x_4) = w$$

$$x_1, x_2, x_3, x_4 \geq 0 \quad w \geq 0$$

1 Sunco Oil has three different processes that can be used to manufacture various types of gasoline. Each process involves blending oils in the company's catalytic cracker. Running process 1 for an hour costs $5 and requires 2 barrels of crude oil 1 and 3 barrels of crude oil 2. The output from running process 1 for an hour is 2 barrels of gas 1 and 1 barrel of gas 2. Running process 2 for an hour costs $4 and requires 1 barrel of crude 1 and 3 barrels of crude 2. The output from running process 2 for an hour is 3 barrels of gas 2. Running process 3 for an hour costs $1 and requires 2 barrels of crude 2 and 3 barrels of gas 2. The output from running process 3 for an hour is 2 barrels of gas 3. Each week, 200 barrels of crude 1, at $2/barrel, and 300 barrels of crude 2, at $3/barrel, may be purchased. All gas produced can be sold at the following per-barrel prices: gas 1, $9; gas 2, $10; gas 3, $24. Formulate an LP whose solution will maximize revenues less costs. Assume that only 100 hours of time on the catalytic cracker are available each week.

1) $t_1$: Process 1 running hours   $y_1$: Gasoline 1 Sold
$t_2$: Process 2 running hours   $y_2$: Gasoline 2 Sold
$t_3$: Process 3 running hours   $y_3$: Gasoline 3 Sold

$x_1$: Crude oil 1 purchased
$x_2$: Crude Oil 2 purchased

| | Cost | Required: ($x_1$) Oil 1 | Required: ($x_2$) Oil 2 | Production: ($y_1$) Gas 1 | Production: ($y_2$) Gas 2 | Production: ($y_3$) Gas 3 |
|---|---|---|---|---|---|---|
| ($t_1$) Production 1 | 5\$ | 2 | 3 | 2 | 1 | – |
| ($t_2$) Production 2 | 4\$ | 1 | 3 | – | 3 | – |

| | Cost | Required: Oil 1 | Required: Oil 2 | Required: Gas 1 | Required: Gas 2 | Production: Gas 3 |
|---|---|---|---|---|---|---|
| ($t_3$) Production 3 | 1\$ | – | 2 | – | 3 | 2 |
| AVAILABLE | | 200 | 300 | PRICE 9\$ | 10\$ | 24\$ |
| COST | | 2\$ | 3\$ | | | |

AVAILABLE TIME = 100 hrs

Maximize $z = 9y_1 + 10y_2 + 24y_3 - 5t_1 - 4t_2 - t_3 - 2x_1 - 2x_2$

Subject to

$t_1 + t_2 + t_3 \leq 100$   $2t_1 + t_2 = x_1$

$x_1 \leq 200$   $3t_1 + 3t_2 + 2t_3 = x_2$

$x_2 \leq 300$   $y_1 = 2t_1$

$y_2 = t_1 + 3t_2 - 2t_3$

$y_3 = 2t_3$

$t_i, x_i, y_i \geq 0$



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**4** Chemco produces three products: 1, 2, and 3. Each pound of raw material costs \$25. It undergoes processing and yields 3 oz of product 1 and 1 oz of product 2. It costs \$1 and takes 2 hours of labor to process each pound of raw material. Each ounce of product 1 can be used in one of three ways.

It can be sold for \$10/oz.

It can be processed into 1 oz of product 2. This requires 2 hours of labor and costs \$1.

It can be processed into 1 oz of product 3. This requires 3 hours of labor and costs \$2.

Each ounce of product 2 can be used in one of two ways.

It can be sold for \$20/oz.

It can be processed into 1 oz of product 3. This requires 1 hour of labor and costs \$6.

Product 3 is sold for \$30/oz. The maximum number of ounces of each product that can be sold is given in Table 23. A maximum of 25,000 hours of labor are available. Determine how Chemco can maximize profit.

**TABLE 23**

| Product | Oz |
|---|---|
| 1 | 5,000 |
| 2 | 5,000 |
| 3 | 3,000 |

4) Three Products 1,2,3

Raw Material Cost: 25\$/lb

1 lb Raw Material $\xrightarrow[+1\$]{2hr}$ { 3 oz Product 1, 1 oz Product 2 }

Product 1: Demand: 5000
- Sold for 10\$/oz
- $\xrightarrow{2hr + 1\$}$ Product 2
- $\xrightarrow{3hr + 2\$}$ Product 3

Product 2: Demand: 5000
- sold for 20\$/oz
- $\xrightarrow{1hr + 6\$}$ Product 3

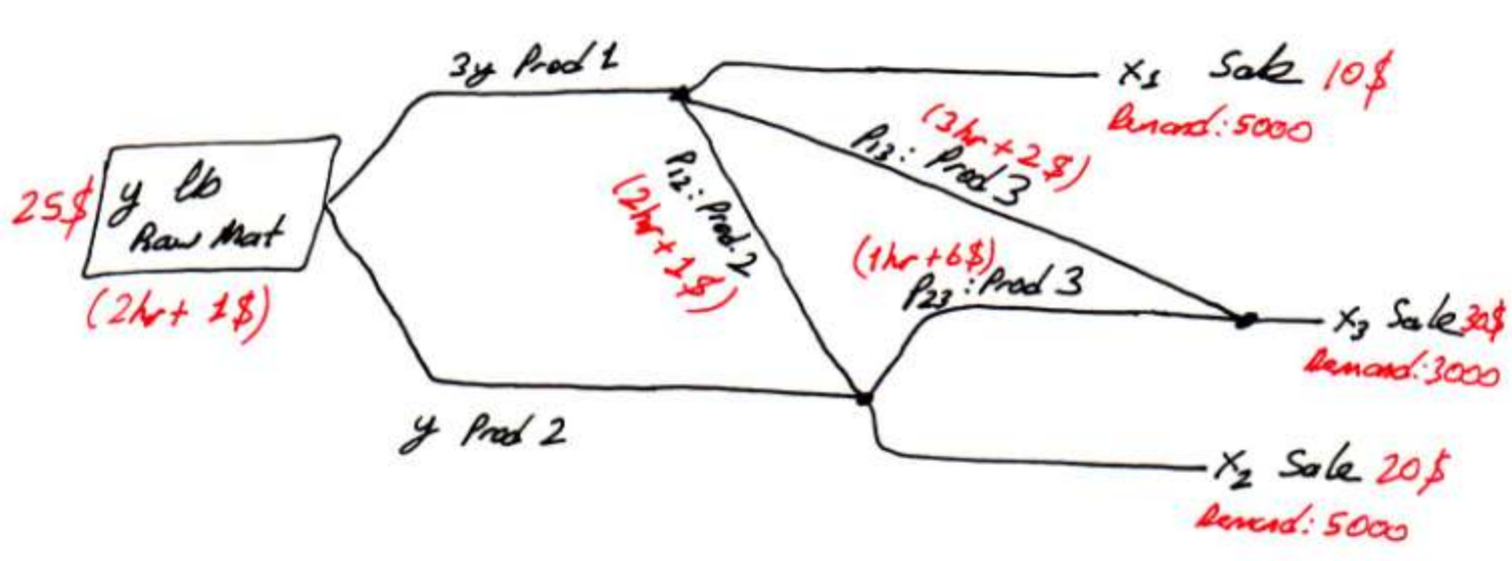
Product 3: Sold for 30\$/oz
Demand: 3000

AVAILABLE HOURS: 25000

y: Raw Material Used

$x_1$: Product 1 Sold
$x_2$: Product 2 Sold
$x_3$: Product 3 Sold

$P_{12}$: Product 1 processed to Product 2
$P_{13}$: Product 1 processed to Product 3
$P_{23}$: Product 2 processed to Product 3



Maximize $z = 10x_1 + 20x_2 + 30x_3 - 25y - y - 2p_{13} - p_{12} - 6p_{23}$

Subject to

$x_1 \le 5000$ $\quad x_1 + p_{12} + p_{13} = 3y$

$x_2 \le 5000$ $\quad p_{12} + y = x_2 + p_{23}$

$x_3 \le 3000$ $\quad x_3 = p_{13} + p_{23}$

$y + 3p_{13} + 2p_{12} + p_{23} \le 25000$ $\quad x_i, p_{ij}, y \ge 0$

## * Inventory Model Problem

**1** A customer requires during the next four months, respectively, 50, 65, 100, and 70 units of a commodity (no backlogging is allowed). Production costs are \$5, \$8, \$4, and \$7 per unit during these months. The storage cost from one month to the next is \$2 per unit (assessed on ending inventory). It is estimated that each unit on hand at the end of month 4 could be sold for \$6. Formulate an LP that will minimize the net cost incurred in meeting the demands of the next four months.

1)

| Period | Demand | Cost |
|---|---|---|
| 1 | 50 | 5\$ |
| 2 | 65 | 8\$ |
| 3 | 100 | 4\$ |
| 4 | 70 | 7\$ |

Storage Cost: 2\$/unit

Salvage at the end of Period 4: 6\$/unit

$x_j$ : Production at period $j$

$i_j$ : Inventory at the end of period $j$

Minimize $z = 5x_1 + 8x_2 + 4x_3 + 7x_4 + 2(i_1 + i_2 + i_3 + i_4) - 6i_4$

Subject to: Period

(1) $x_1 - i_1 = 50$

(2) $i_1 + x_2 - i_2 = 65$

(3) $i_2 + x_3 - i_3 = 100$

(4) $i_3 + x_4 - i_4 = 70$

$x_j, i_j \ge 0 \quad j = 1, 2, 3, 4$

**2** A company faces the following demands during the next three periods: period 1, 20 units; period 2, 10 units; period 3, 15 units. The unit production cost during each period is as follows: period 1—\$13; period 2—\$14; period 3—\$15. A holding cost of \$2 per unit is assessed against each period's ending inventory. At the beginning of period 1, the company has 5 units on hand.

In reality, not all goods produced during a month can be used to meet the current month's demand. To model this fact, we assume that only one half of the goods produced during a period can be used to meet the current period's demands. Formulate an LP to minimize the cost of meeting the demand for the next three periods. (*Hint:* Constraints such as $i_1 = x_1 + 5 - 20$ are certainly needed. Unlike our example, however, the constraint $i_1 \geq 0$ will not ensure that period 1's demand is met. For example, if $x_1 = 20$, then $i_1 \geq 0$ will hold, but because only $\frac{1}{2}(20) = 10$ units of period 1 production can be used to meet period 1's demand, $x_1 = 20$ would not be feasible. Try to think of a type of constraint that will ensure that what is available to meet each period's demand is at least as large as that period's demand.)

2)

| Period | Demand | Cost |
|---|---|---|
| 1 | 20 | 13 |
| 2 | 10 | 14 |
| 3 | 15 | 15 |

Holding Cost: 2\$/units

Beginning Inventory: 5 units

$x_j$: Production at period $j$

$i_j$: Inventory at the end of period $j$

Minimize $z = 13x_1 + 14x_2 + 15x_3 + 2(i_1 + i_2 + i_3)$

Subject to:

Period

(1) $5 + x_1 - i_1 = 20$ $\quad i_1 \geq 20$

(2) $i_1 + x_2 - i_2 = 10$ $\quad i_1 - i_2 \geq 10$

(3) $i_2 + x_3 - i_3 = 15$ $\quad i_2 - i_3 \geq 15$ $\quad x_j, i_j \geq 0 \quad j = 1,2,3$

**3** James Beerd bakes cheesecakes and Black Forest cakes. During any month, he can bake at most 65 cakes. The costs per cake and the demands for cakes, which must be met on time, are listed in Table 33. It costs 50¢ to hold a cheesecake, and 40¢ to hold a Black Forest cake, in inventory for a month. Formulate an LP to minimize the total cost of meeting the next three months' demands.

**TABLE 33**

| Item | Month 1 | | Month 2 | | Month 3 | | Holding Cost |
|---|---|---|---|---|---|---|---|
| | Demand | Cost/Cake (\$) | Demand | Cost/Cake (\$) | Demand | Cost/Cake (\$) | |
| Cheesecake | 40 | 3.00 | 30 | 3.40 | 20 | 3.80 | 0,5 \$ |
| Black Forest | 20 | 2.50 | 30 | 2.80 | 10 | 3.40 | 0,4 \$ |

$x_j$: Cheesecake Baked at month $j$

$c_j$: Cheesecake at the end at month $j$ in the inventory

$y_j$: BlackForest Baked at month $j$

$b_j$: BlackForest at the end at month $j$ in the inventory

AVAILABLE BAKE = 65

Minimize $z = 3x_1 + 3{,}4x_2 + 3{,}8x_3 + 0{,}5(c_1 + c_2 + c_3)$
$+ 2{,}5y_1 + 2{,}8y_2 + 3{,}4y_3 + 0{,}4(b_1 + b_2 + b_3)$

Subject to;

| | | |
|---|---|---|
| $x_1 - c_1 = 40$ | $y_1 - b_1 = 20$ | $x_1 + y_1 \leq 65$ |
| $c_1 + x_2 - c_2 = 30$ | $b_1 + y_2 - b_2 = 30$ | $x_2 + y_2 \leq 65$ |
| $c_2 + x_3 - c_3 = 20$ | $b_2 + y_3 - b_3 = 10$ | $x_3 + y_3 \leq 65$ |

$x_j, y_j, c_j, b_j \geq 0 \quad j = 1,2,3$

**8** A company must meet (on time) the following demands: quarter 1—30 units; quarter 2—20 units; quarter 3—40 units. Each quarter, up to 27 units can be produced with regular-time labor, at a cost of \$40 per unit. During each quarter, an unlimited number of units can be produced with overtime labor, at a cost of \$60 per unit. Of all units produced, 20% are unsuitable and cannot be used to meet demand. Also, at the end of each quarter, 10% of all units on hand spoil and cannot be used to meet <u>any future demands</u>. After each quarter's demand is satisfied and spoilage is accounted for, a cost of \$15 per unit is assessed against the quarter's ending inventory. Formulate an LP that can be used to minimize the total cost of meeting the next three quarters' demands. Assume that 20 usable units are available at the beginning of quarter 1.

| Period | Demand | | Cost |
|---|---|---|---|
| 1 | 30 | Regular Time | 40\$/unit |
| 2 | 20 | OverTime | 60\$/unit |
| 3 | 40 | Inventory | 15\$/unit |

$I_0 = 20$ / Reg. Time: Max. 27 units

$r_j$: Regular time Production at period $j$
$o_j$: Overtime Production at period $j$
$i_j$: Ending inventory of period $j$
$L_j$: Inventory left available for future demands

Minimize $z = 40(r_1 + r_2 + r_3) + 60(o_1 + o_2 + o_3) + 15(i_1 + i_2 + i_3)$

Subject to:

Period

(1) $20 + 0{,}80(r_1 + o_1) - i_1 = 30$
$L_1 = 0{,}90 \cdot [\, i_1 + 0{,}20 \cdot (r_1 + o_1)]$

(2) $L_1 + 0{,}80 \cdot (r_2 + o_2) - i_2 = 20$
$L_2 = 0{,}90 \cdot [\, i_2 + 0{,}20 \cdot (r_2 + o_2)]$

(3) $L_2 + 0{,}80\,(r_3 + o_3) - i_3 = 40$
$L_3 = 0{,}90 \cdot [\, i_3 + 0{,}20 \cdot (r_3 + o_3)]$

$r_j, o_j, i_j, L_j \geq 0$
$j = 1,2,3$

# Multiperiod Financial Models

3 At time 0, we have \$10,000. Investments A and B are available; their cash flows are shown in Table 39. Assume that any money not invested in A or B earns *no* interest. Formulate an LP that will maximize cash on hand at time 3. Can you guess the optimal solution to this problem?

TABLE 39

| Time | A | B |
|---|---|---|
| 0 | −\$1 | \$0 |
| 1 | \$0.2 | −\$1 |
| 2 | \$1.5 | \$0 |
| 3 | \$0 | \$1.9 |

3) A: Investment to A, B: Investment to B

$C_j$: Cash uninvested at time $j$: $j = 0, 1, 2$

Maximize $z = C_2 + 1{,}9B$

Subject to: Time Available = Invested + Uninvested

(0) $10000 = A + C_0$

(1) $C_0 + 0{,}2A = B + C_1$

(2) $C_1 + 1{,}5 = C_2$ $\qquad A, B, C_j \geq 0$

8 I now have \$100. The following investments are available during the next three years:

**Investment A** Every dollar invested now yields \$0.10 a year from now and \$1.30 three years from now.

**Investment B** Every dollar invested now yields \$0.20 a year from now and \$1.10 two years from now.

**Investment C** Every dollar invested a year from now yields \$1.50 three years from now.

During each year, uninvested cash can be placed in money market funds, which yield 6% interest per year. At most \$50 may be placed in each of investments A, B, and C. Formulate an LP to maximize my cash on hand three years from now.

8)

| Time | Inv A | Inv B | Inv C |
|---|---|---|---|
| 0 | −1 | −1 | −1 |
| 1 | 0,1 | 0,2 | 0 |
| 2 | 0 | 1,1 | 0 |
| 3 | 1,3 | 0 | 1,5 |

Interest → 6% /year

Each investment ≤ 50

A, B, C → Investment to A, B, C

$S_t$: Cash placed in money Market funds at time 0, 1, 2

Maximize $z = 1{,}3A + 1{,}5C + 1{,}06S_2$

Subject to: Time Available = Invested + Market funds

(0) $100 = A + B + C + S_0$

(1) $1{,}06S_0 + 0{,}1A + 0{,}2B = S_1$

(2) $1{,}06S_1 + 1{,}1B = S_2$

$A, B, C \geq 0 \quad S_j \geq 0 \quad j = 0, 1, 2$

30

TABLE 44

| Time (Years) | Cash Flow Project 1 | Project 2 | Project 3 |
|---|---|---|---|
| 0 | −3 | −2 | −2 |
| .5 | −1 | −.5 | −2 |
| 1 | +1.8 | 1.5 | −1.8 |
| 1.5 | 1.4 | 1.5 | 1 |
| 2 | 1.8 | 1.5 | 1 |
| 2.5 | 1.8 | .2 | 1 |
| 3 | 5.5 | −1 | 6 |

**9** Winstonco is considering investing in three projects. If we fully invest in a project, the realized cash flows (in millions of dollars) will be as shown in Table 44. For example, project 1 requires cash outflow of \$3 million today and returns \$5.5 million 3 years from now. Today we have \$2 million in cash. At each time point (0, .5, 1, 1.5, 2, and 2.5 years from today) we may, if desired, borrow up to \$2 million at 3.5% (per 6 months) interest. Leftover cash earns 3% (per 6 months) interest. For example, if after borrowing and investing at time 0 we have \$1 million we would receive \$30,000 in interest at time .5 years. Winstonco's goal is to maximize cash on hand after it accounts for time 3 cash flows. What investment and borrowing strategy should be used? Remember that we may invest in a fraction of a project. For example, if we invest in .5 of project 3, then we have cash outflows of −\$1 million at time 0 and .5.

9) $p_j$ : Fraction of the project $j$ invested, $j = 1, 2, 3$

$b_i$ : Money Borrowed at time $i \in \{00, 05, 10, 15, 20, 25\}$

$c_i$ : Leftover cash at time $i$ — Borrow Rate: 3,5% ; Leftover Rate: 3%

Maximize $z = 5,5p_1 - 1.p_2 + 6.p_3 + 1,03c_{25} - 1,035b_{25}$

Subject to: Time — AVAILABLE = Invested + Borrowed + Notinvested

$$00 \qquad 3 + b_{00} = 3p_1 + 2p_2 + 2p_3 + c_{00}$$

$$05 \qquad 1,03c_{00} + b_{05} = 1p_1 + 0,5p_2 + 2p_3 + 1,035b_{00} + c_{05}$$

$$10 \qquad 1,03c_{05} + b_{10} + 1,8p_1 + 1,5p_2 = 1,8p_3 + 1,035b_{05} + c_{10}$$

$$15 \qquad 1,03c_{10} + b_{15} + 1,4p_1 + 1,5p_2 + 1p_3 = 1,035b_{10} + c_{15}$$

$$20 \qquad 1,03c_{15} + b_{20} + 1,8p_1 + 1,5p_2 + 1p_3 = 1,035b_{15} + c_{20}$$

$$25 \qquad 1,03c_{20} + b_{25} + 5,5p_1 + 6p_3 = 1,035b_{20} + c_{25}$$

$$p_1 \le 1$$
$$p_2 \le 1$$
$$p_3 \le 1$$

$p_j \ge 0 \quad j = 1, 2, 3 \; ; \; b_i, c_i \ge 0 \quad i \in \{00, 05, 10, 15, 20, 25\}$



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# * Multiperiod Work Scheduling

**2** An insurance company believes that it will require the following numbers of personal computers during the next six months: January, 9; February, 5; March, 7; April, 9; May, 10; June, 5. Computers can be rented for a period of one, two, or three months at the following unit rates: one-month rate, \$200; two-month rate, \$350; three-month rate, \$450. Formulate an LP that can be used to minimize the cost of renting the required computers. You may assume that if a machine is rented for a period of time extending beyond June, the cost of the rental should be prorated. For example, if a computer is rented for three months at the beginning of May, then a rental fee of $\frac{2}{3}(450) = \$300$, not \$450, should be assessed in the objective function.

2)

| Rented Time of Comp. / Month | (200) 1 | (350) 2 | (450) 3 | # of Comp. Required |
|---|---|---|---|---|
| 1 | $x_{11}$ | $x_{12}$ | $x_{13}$ | 9 |
| 2 | $x_{21}$ | $x_{22}$ | $x_{23}$ | 5 |
| 3 | $x_{31}$ | $x_{32}$ | $x_{33}$ | 7 |
| 4 | $x_{41}$ | $x_{42}$ | $x_{43}$ | 9 |
| 5 | $x_{51}$ | $x_{52}$ | $x_{53}$ | 10 |
| 6 | $x_{61}$ | $x_{62}$ | $x_{63}$ | 5 |

| Rent Time | Cost |
|---|---|
| 1 | 200\$ |
| 2 | 350\$ |
| 3 | 450\$ |

minimize $z = 200.(x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61})$
$+350(x_{12} + x_{22} + x_{32} + x_{42} + x_{52}) + 175x_{62} + 450(x_{13} + x_{23} + x_{33} + x_{43}) + 300x_{53} + 150x_{63}$

Subject to: Month

(1) $x_{11} + x_{12} + x_{13} \geq 9$

(2) $x_{21} + x_{12} + x_{22} + x_{13} + x_{23} \geq 5$

(3) $x_{31} + x_{22} + x_{32} + x_{13} + x_{23} + x_{33} \geq 7$

(4) $x_{41} + x_{32} + x_{42} + x_{23} + x_{33} + x_{43} \geq 9$

(5) $x_{51} + x_{42} + x_{52} + x_{33} + x_{43} + x_{53} \geq 10$

(6) $x_{61} + x_{52} + x_{62} + x_{43} + x_{53} + x_{63} \geq 5$

$x_{ij} \geq 0$ for $i = 1,2,3,4,5,6$
$j = 1,2,3$



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TABLE 46

| Month | Selling Price ($) | Purchase Price ($) |
|---|---|---|
| 1 | 3 | 8 |
| 2 | 6 | 8 |
| 3 | 7 | 2 |
| 4 | 1 | 3 |
| 5 | 4 | 4 |
| 6 | 5 | 3 |
| 7 | 5 | 3 |
| 8 | 1 | 2 |
| 9 | 3 | 5 |
| 10 | 2 | 5 |

**4** You own a wheat warehouse with a capacity of 20,000 bushels. At the beginning of month 1, you have 6,000 bushels of wheat. Each month, wheat can be bought and sold at the price per 1000 bushels given in Table 46.

The sequence of events during each month is as follows:

**a** You observe your initial stock of wheat.

**b** You can sell any amount of wheat up to your initial stock at the current month's selling price.

**c** You can buy (at the current month's buying price) as much wheat as you want, subject to the warehouse size limitation.

Your goal is to formulate an LP that can be used to determine how to maximize the profit earned over the next 10 months.

4) $x_i$: # of 1000 buskels at the beginning of month $i$

$s_i$: # of 1000 bushels sold at ~~the beginning of~~ month $i$

$p_i$: # of 1000 buskels purchased at month $i$

Maximize $z = 3s_1 + 6s_2 + \cdots + 2s_{10} - 8p_1 - 8p_2 - \cdots - 5p_{10}$

Subject to:

$x_1 = 6$ $\quad s_1 \le x_1$

$x_2 = 6 - s_1 + p_1$ $\quad s_2 \le x_2$

$x_3 = x_2 - s_2 + p_2$ $\quad s_3 \le x_3$

$\vdots$ $\quad \vdots$ $\quad s_i, p_i, x_i \ge 0$

$x_{10} = x_9 - s_9 + p_9$ $\quad s_{10} \le x_{10}$ $\quad i = 1, 2, \ldots, 10$

**58**[‡] To process income tax forms, the IRS first sends each form through the data preparation (DP) department, where information is coded for computer entry. Then the form is sent to data entry (DE), where it is entered into the computer. During the next three weeks, the following number of forms will arrive: week 1, 40,000; week 2, 30,000; week 3, 60,000. The IRS meets the crunch by hiring employees who work 40 hours per week and are paid $200 per week. Data preparation of a form requires 15 minutes, and data entry of a form requires 10 minutes. Each week, an employee is assigned to either data entry or data preparation. The IRS must complete processing of all forms by the end of week 5 and wants to minimize the cost of accomplishing this goal. Formulate an LP that will determine how many workers should be working each week and how the workers should be assigned over the next five weeks.

58) $x_i$: # of workers work for DP dept. at week $i = 1, 2, 3, 4, 5$

$y_i$: # of workers work for DE dept. at week $i$

$p_i$: Work left at DP dept at the end of week $i$

$e_i$: Work left at DE dept. at the end of week $i$

1 employee : 40 hr/week : Cost 200\$ / week.

1 employee can finish $\frac{40 \cdot 60}{15} = 160$ works in DP dept.

and $\frac{40 \cdot 60}{10} = 240$ works in DE dept.

| Week | # of forms |
|---|---|
| 1 | 40000 |
| 2 | 30 000 |
| 3 | 60 000 |

$$\text{Minimize} \quad z = 200 \cdot \left( \sum_{i=1}^{5} x_i + \sum_{i=1}^{5} y_i \right)$$

subject to:

| Weeks | work left at DP | work left at DE |
|---|---|---|
| (1) | $p_1 = 40000 - 160x_1$ | $e_1 = 40000 - 240y_1$ |
| (2) | $p_2 = p_1 - 160x_2 + 30000$ | $e_2 = e_1 - 240y_2 + 30000$ |
| (3) | $p_3 = p_2 - 160x_3 + 60000$ | $e_3 = e_2 - 240y_3 + 60000$ |
| (4) | $p_4 = p_3 - 160x_4$ | $e_4 = e_3 - 240y_4$ |
| (5) | $p_5 = p_4 - 160x_5$ | $e_5 = e_4 - 240y_5$ |

$e_1 \geq p_1$ $\qquad e_5 = 0$ $\qquad e_i, p_i, x_i, y_i \geq 0$

$e_2 \geq p_2$ $\qquad p_5 = 0$ $\qquad i = 1, 2, 3, 4, 5$

$e_3 \geq p_3$

$e_4 \geq p_4$